Lesson 4:

Limit Equilibrium Framework for Internal Stability at Limit State

Prepared and Presented by Dov Leshchinsky.

Virtual Presentation was in Geosynthetics 2021 as one of three permissible design methods in AASHTO 2020.

This presentation serves as a background for ReSSA+

https://www.fhwa.dot.gov/engineering/geotech/pubs/hif17004.pdf

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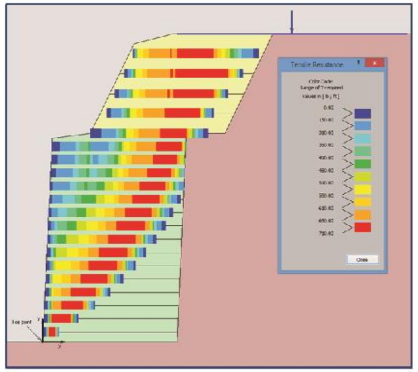

LIMIT EQUILIBRIUM DESIGN FRAMEWORK FOR MSE STRUCTURES WITH EXTENSIBLE REINFORCEMENT

1. Provides analytical details

2. Verifies using numerical and physical models

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16. Abstract

Current design of reinforced soil structures in the U.S. distinguishes between slopes and walls using the batter angle as a criterion. Using a unified approach in limit state design of reinforced 'walls' and 'slopes' should diminish confusion while enabling a wide and consistent usage in solving geotechnical problems such as complex geometries and soil profiles. Limit equilibrium (LE) analysis has been used successfully in the design of complex and critical (e.g., tall dams) for many decades. Limit state analysis, including LE, assumes that the *design* strength of the soil is mobilized. Presented is a LE framework, limited to extensible reinforcement, which enables the designer to find the tensile force distribution in each layer required at a limit state. This approach is restricted to Allowable Stress Design (ASD). Three example problems are presented.

17. Key Words	18. Distribution Statement
Mechanically Stabilized Earth Wall Design, MSE Wall Design, Limit Equilibrium, Geotechnical, Extensible reinforcement	No restrictions.

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Roadmap of Presentation

- **Why Limit State analysis is needed?**
- Available Limit State Methods of Analysis
- Limit Equilibrium: Global Approach
- The Safety Map Tool
- Limit Equilibrium: Baseline Solution
- Limit Equilibrium: Design Approach
- Limit Equilibrium: Design Examples
- Concluding Remarks

Why Check Against Limit State?

- Limit State failure is a realistic possibility
- Such state is avoided by assigning adequate margins of safety in design
- To quantify margins of safety, one needs to reliably predict limit state conditions
- Theoretically, design considering serviceability alone should eliminate possible limit state
- However, predicting displacements in practice is poor whereas predicting failure is quite reliable

∴ **Practice:** Design for limit state using adequate margins of safety implicitly satisfies serviceability

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Limit State: ***Analysis*** **and** ***Design***

- Premise of Limit State ***Analysis***:
  - Failure is imminent
  - Strength of all elements resisting failure are mobilized simultaneously
  - ***Analysis*** → ***Stage I*** → Calculate $T_{max}(Z)$, $T_o(Z)$ → aka *Internal Stability*
- Premise of Limit State ***Design***:
  - Developed 'active wedge' is resisted by reinforcement → Select reinforcement with adequate margin of safety against rupture
  - Ensure existence of margin of safety on strength of soil
  - ***Design*** → ***Stage II*** → Use LTDS to assess *Fs* on soil strength

➢ ***Analysis*** is the basis for ***Design*** → It defines conditions for imminent failure → Allows for meaningful use of prescribed margins of safety on strength of both soil and geosynthetic

Limit State Analysis: Lateral Earth Pressure - Simplified AASHTO

- Semi-empirical, verified at working load conditions (i.e., not at limit state)
- Safe, fortunately economical, and easy to use➔ *Credit*: Turned an innovative technology into a commodity
- Batter is limited to ≤20° ➔ What about slopes?
- What about complex geometries? Extrapolation to realistic geotechnical conditions (e.g., variable layout of reinforcement, marginal soils, tiered slopes/walls)?
- Is it actually adequate for limit state? If not, could it partially be overly-conservative?

Limit State Analysis: Continuum Mechanics (FE, FD)

- Comprehensive approach
- Valid for walls and slopes
- More complicated than AASHTO ➔ Could be useful in identifying potential failure geometries in complex problems
- Not yet a common design tool in the US
- Impractical tool to generate the *instructive* Tension Map at limit state (i.e., baseline solution explained later)

Limit State Analysis: Global Limit Equilibrium (LE)

- Simple and yet applicable to complex problems
- No arbitrary distinction between ‘wall’ and ‘slope’
- Global LE design is half-cooked➔ Strength is examined globally - along a singular slip surface - while locally **required** strength, including connections, is overlooked ➔ That is, it ignores local demand by smearing (shedding or averaging) the load over all layers

∴Does not deal explicitly with ‘Internal Stability’ which is concerned with local demand ➔ It provides an important, but narrow, design perspective ➔ It skips ***Stage I***

LE is Classic...

- Coulomb in 1766: Resultant force on a retaining is based on LE approach
- First formulation related to slope stability: Culmann Wedge in 1866

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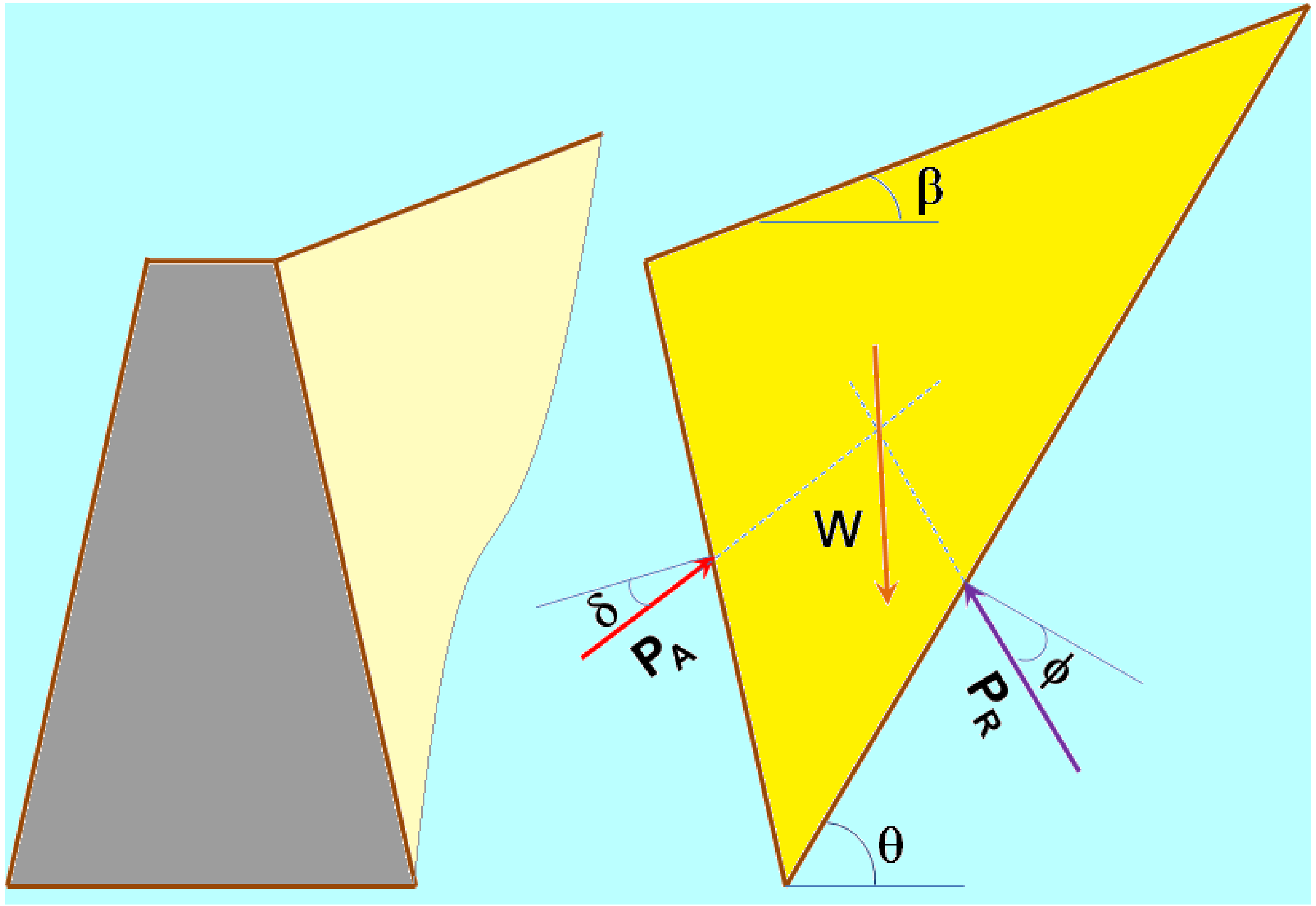
Coulomb (1776) Active Wedge – Gravity Wall: Find max(P_A)

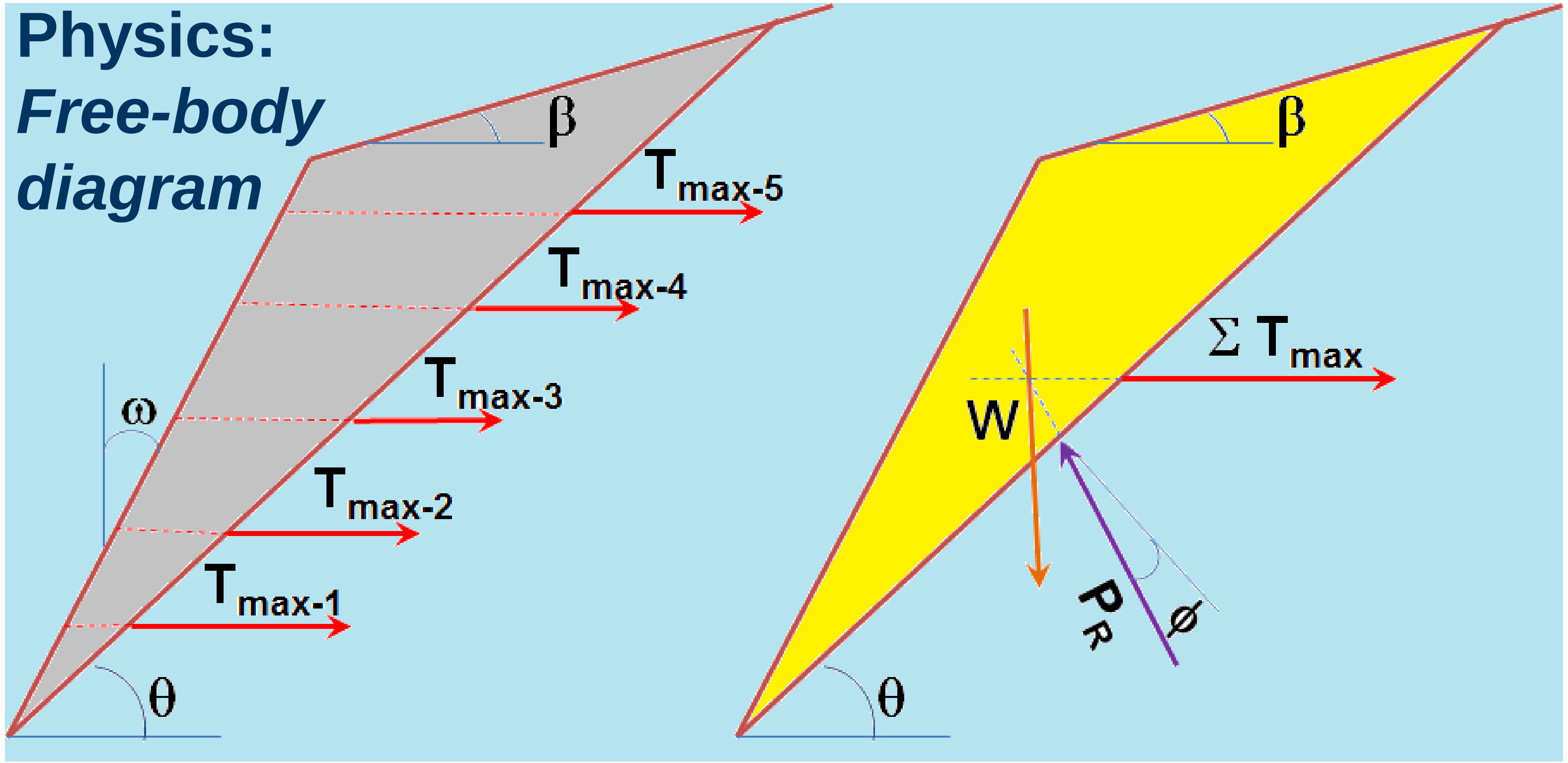


Physics is timeless…
➔ *Free-body diagram yielding equilibrium*

Premise: *Small outwards wall movement* ➔ *Active soil wedge forms* ➔ P_o *drops to* P_A

Note: *Formation of planar surface does not mean wall failure* ➔ *Wall is designed to resist the active wedge*

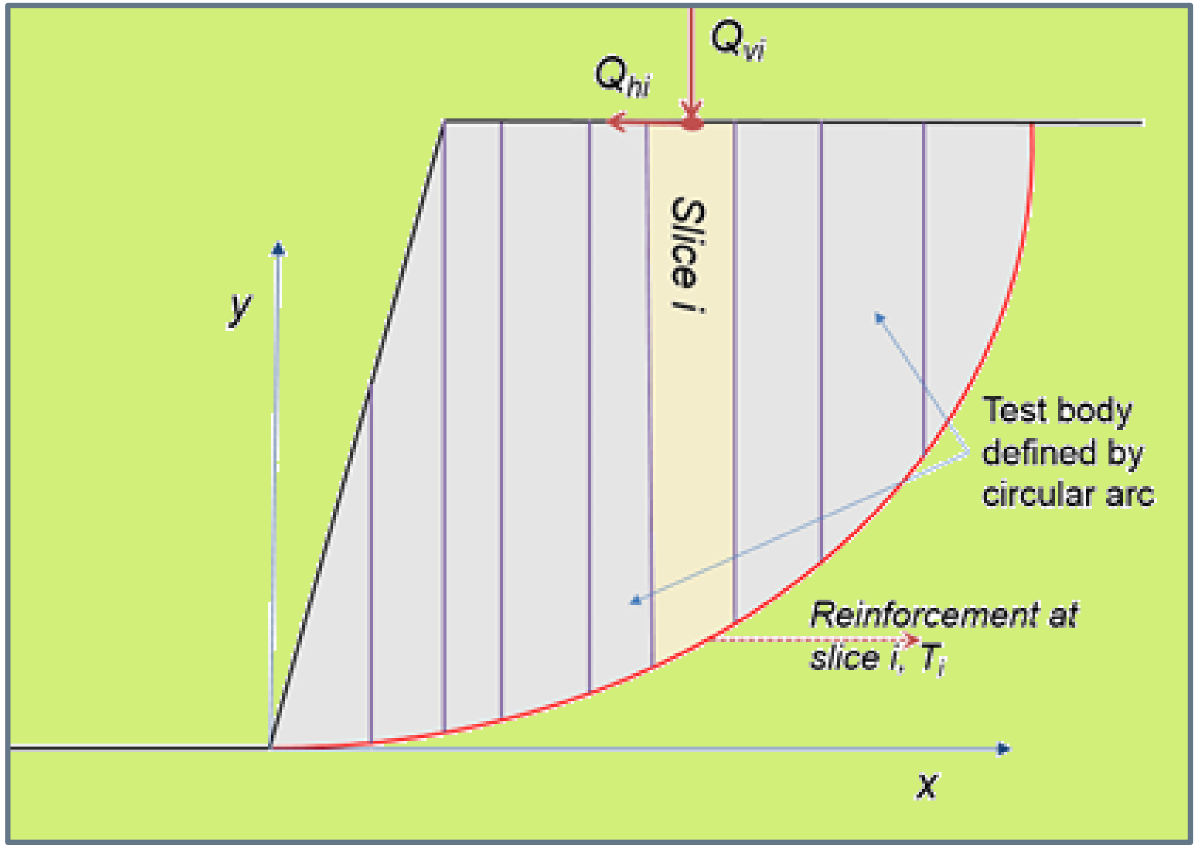
Culmann (1866) Critical Wedge for Reinforced Slope: Find max$\Sigma(T_{max})$

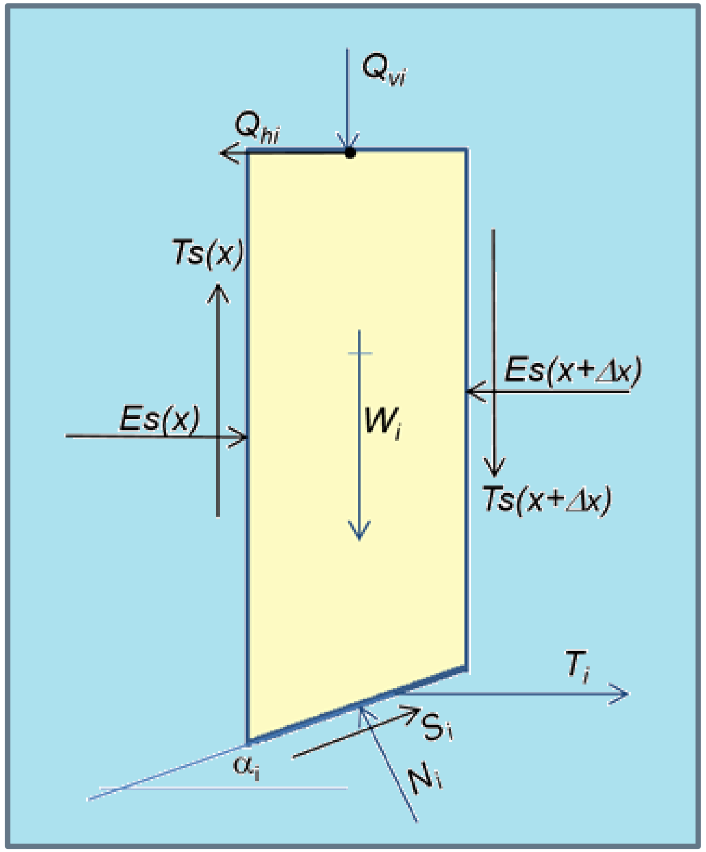


Small stretch of reinforcement ➔ Active wedge develops ➔ Load in reinforcement drops to T_{max}

Note: *Formation of slip surface does not mean structural failure ➔ Reinforcement is designed to resist the active soil wedge*

Bishop (1955) Circular Arc: Find min(SF)=Fs





Bishop considers layered soil/complex problems. Circle can degenerate to planar surface (if it is more critical), however, ***a priori*** *assumed planar surface cannot degenerate to curved surface ➔ Valid for slopes and walls…*

LE: Bishop Basic Formulation

Safety Factor = SF = $\sum M_R / \sum M_D$ (sum is over *n* slices)
Factor of Safety = Fs= min(SF)

$$\boldsymbol{SF} = \frac{\sum \dfrac{c_i' \, \Delta x_i + (W_i + Q_{vi} - u_i \, \Delta x_i) \, tan \;\; \phi_i'}{m_{\alpha i}}}{\sum \left[(W_i + Q_{vi}) \sin \alpha_i \; + \dfrac{Q_{hi} d_i}{R} \; - T_i \; R_c \cos \alpha_i \right]}$$

where $m_{\alpha i} = \cos \alpha_i + (\sin \alpha_i \tan \phi_i')/\boldsymbol{SF}$

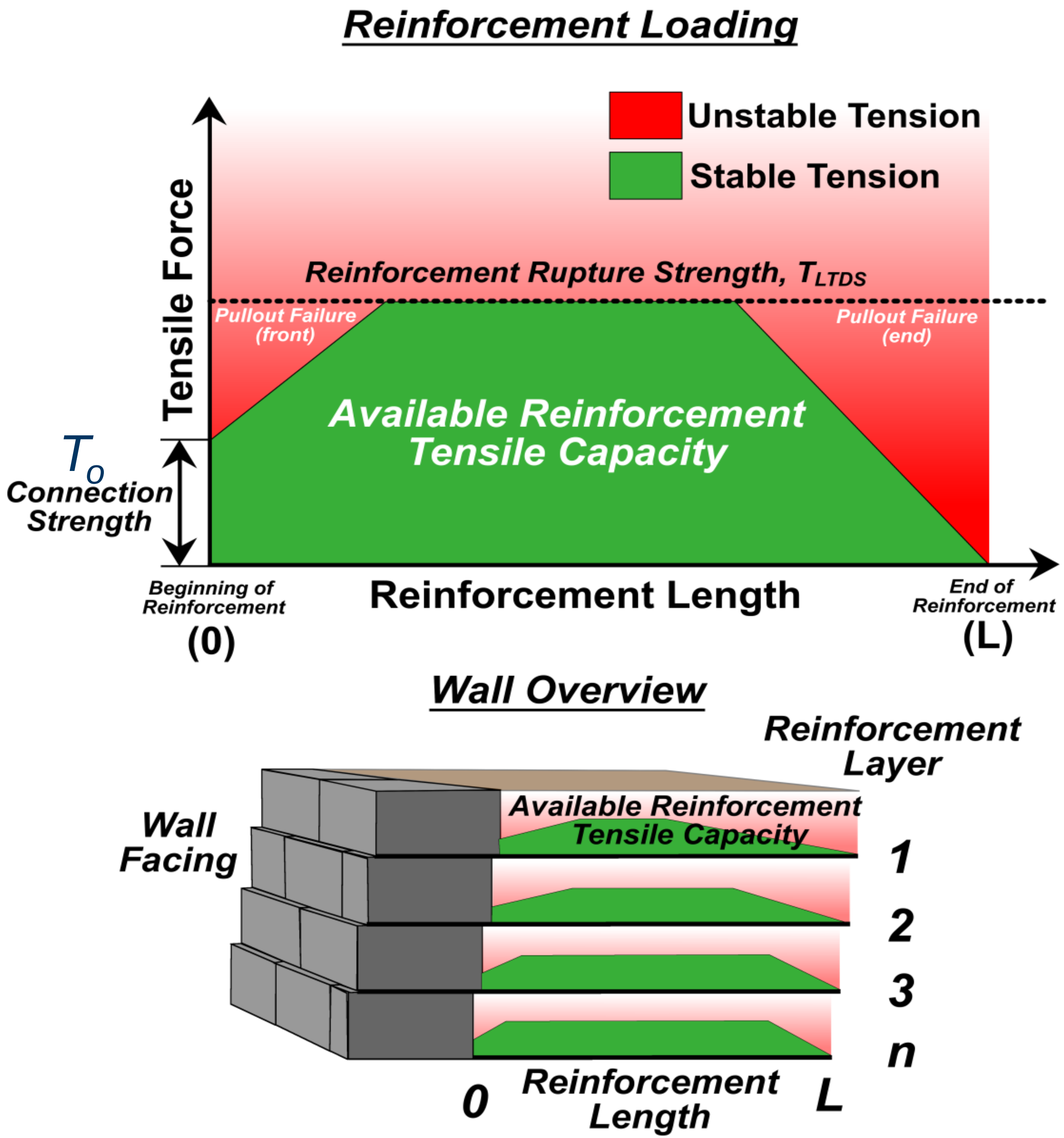
$$\boldsymbol{Fs} = \min(\boldsymbol{SF})$$

T_i is reinforcement force at intersection with surface

LE:

T_i = Tensile Capacity Along Each Layer of Reinforcement

(Note: Front and rear pullout resistance enables the mobilization of LTDS=T_{LTDS})



Can We Use ‘Better’ LE Methods

- Yes, we can…
- Recall the term ***framework*** in the title of presentation – it is ***not*** restricted to a specific method of analysis
- Han and Leshchinsky (2006) used Culmann - instructive but has limited use
- Leshchinsky et al. (2014) used log spiral - rigorous but not easy to use (also, limited to homogenous problems)
- Leshchinsky et al. (2017) used Bishop - not rigorous but practical
- YOU may use rigorous a LE (e.g., Spencer, M-P) with general slip surface. You can even use **Limit Analysis**…

Roadmap of Presentation

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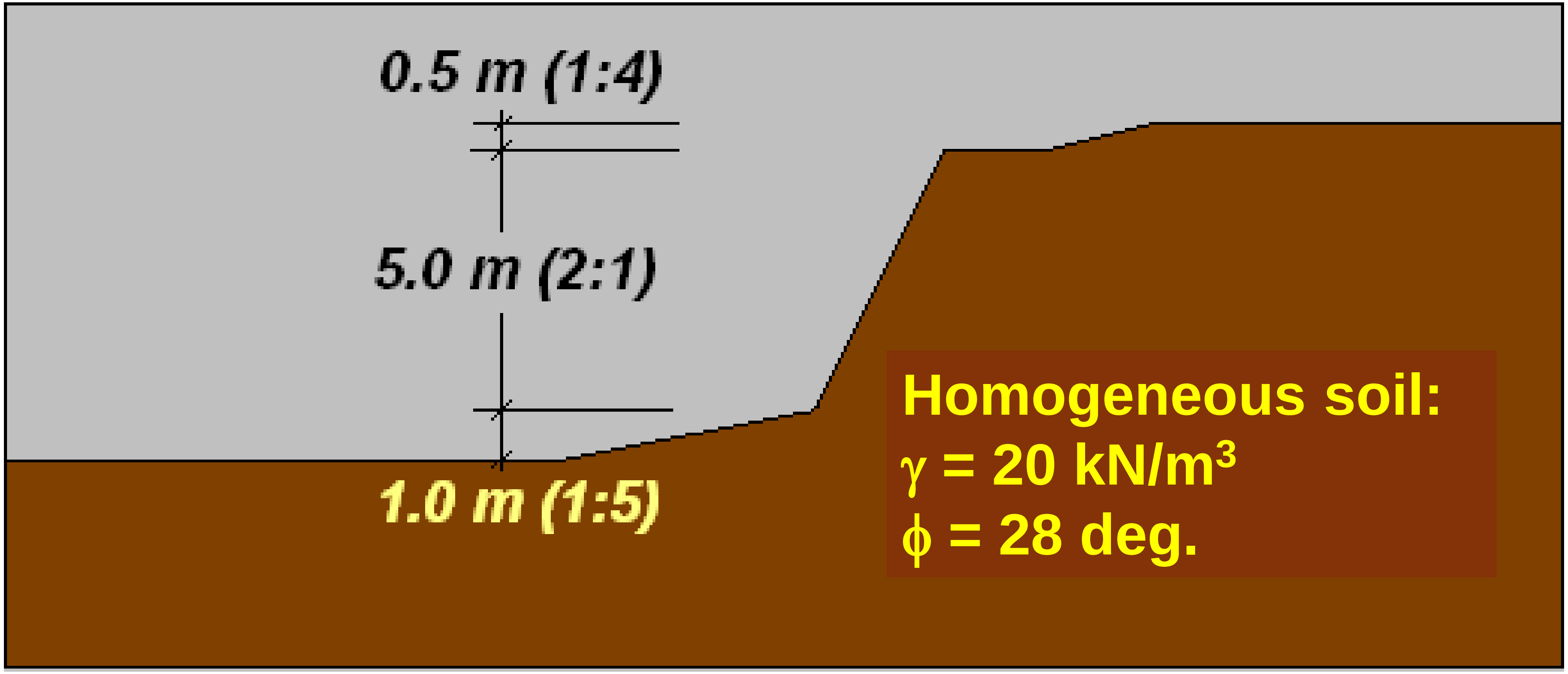
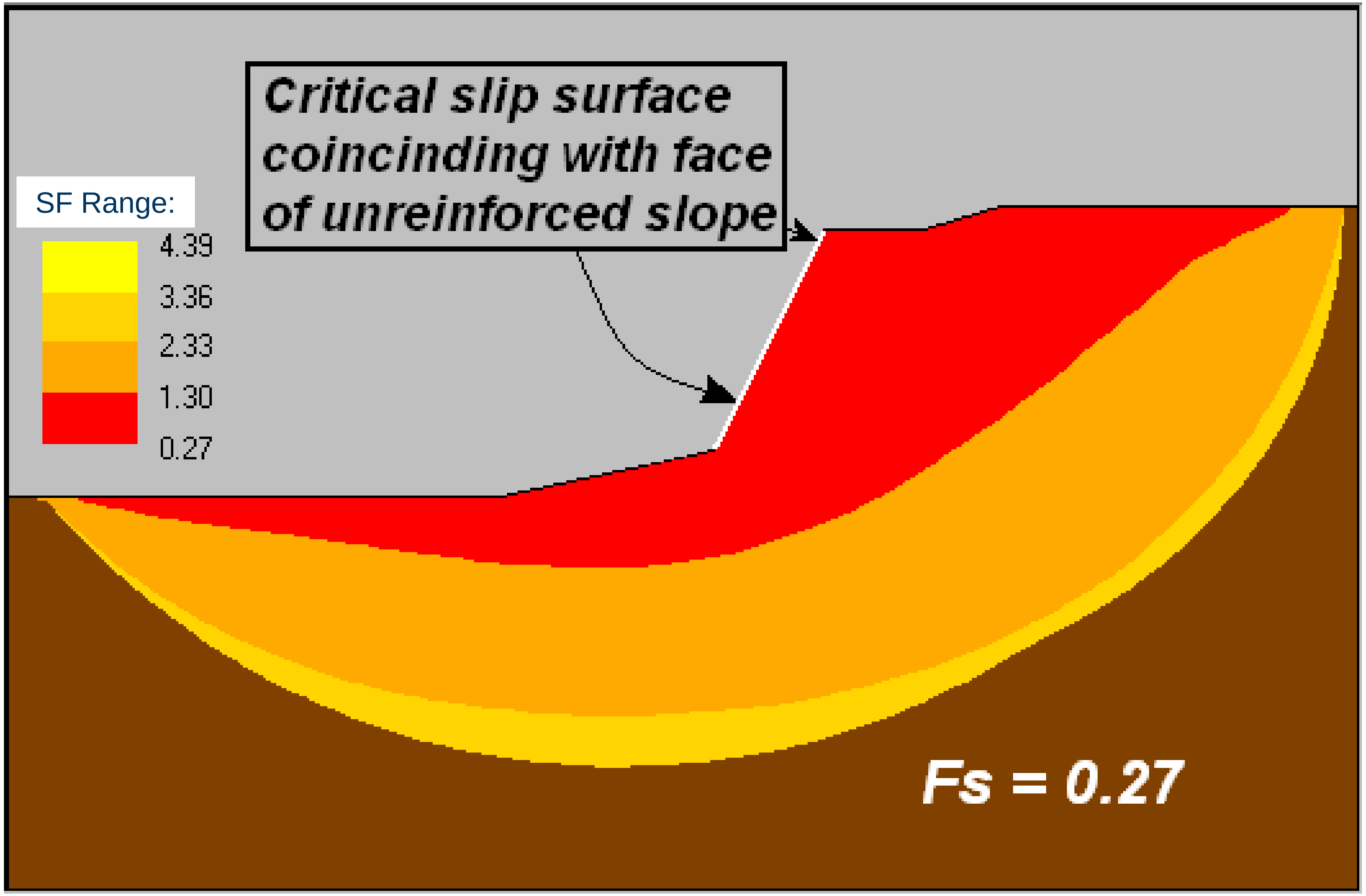
The Safety Map Tool

- **Safety Map:** Baker and Leshchinsky (2001) introduced the concept, proved its mathematical validity, and coined the term
- **Safety Map** = Color-coded map showing the ***spatial*** distribution of the ***safety factors***, SF, in a slope ➔ Visual diagnostic tool for the state of stability of a reinforced mass
- **Design Objective:** Select strength & layout of reinforcement to produce an efficient structure that is adequately stable

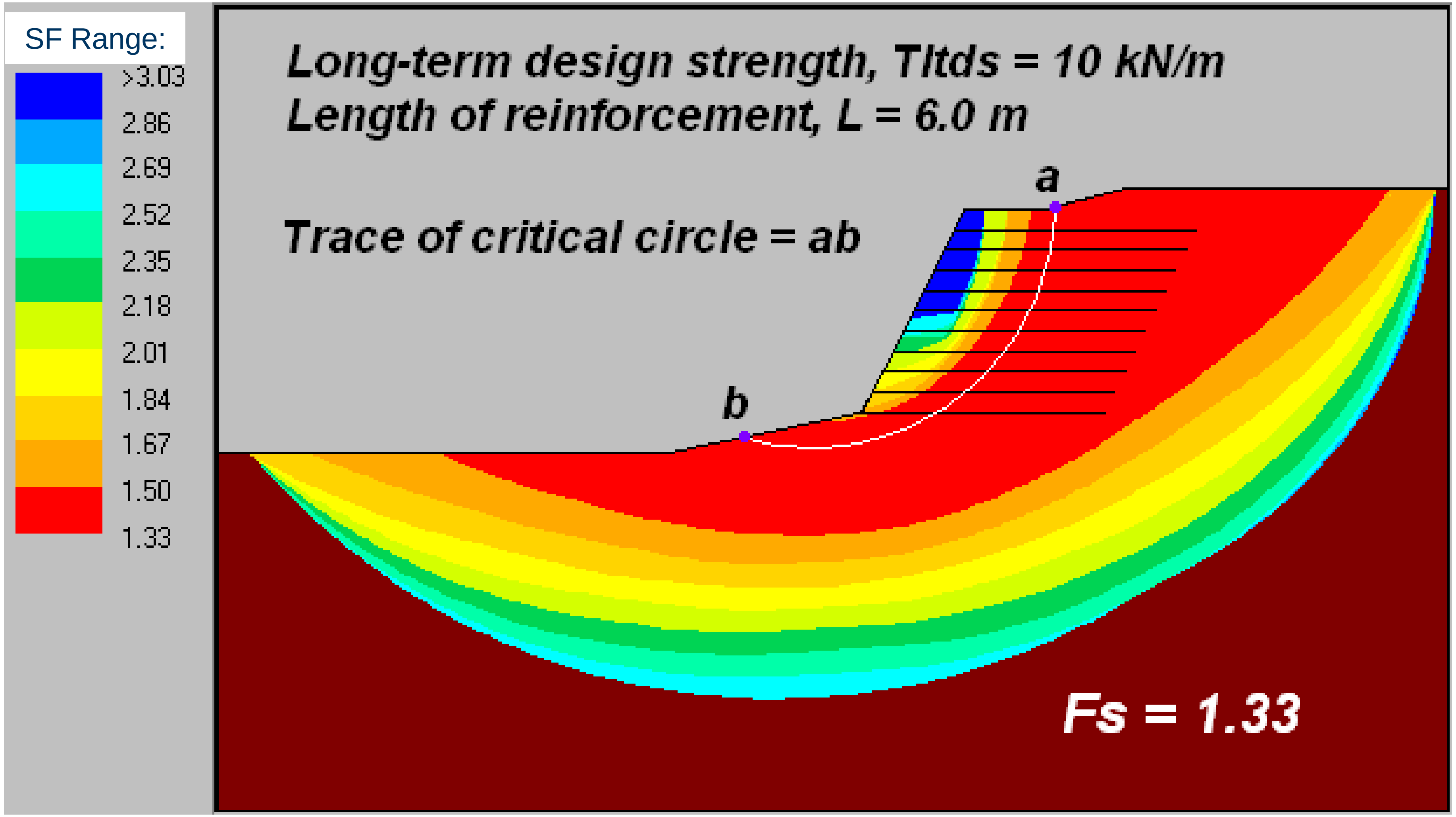
Example Problem



Unreinforced Problem (Bishop)



Adequate Reinforcement Layout using Circular Arc (Bishop)



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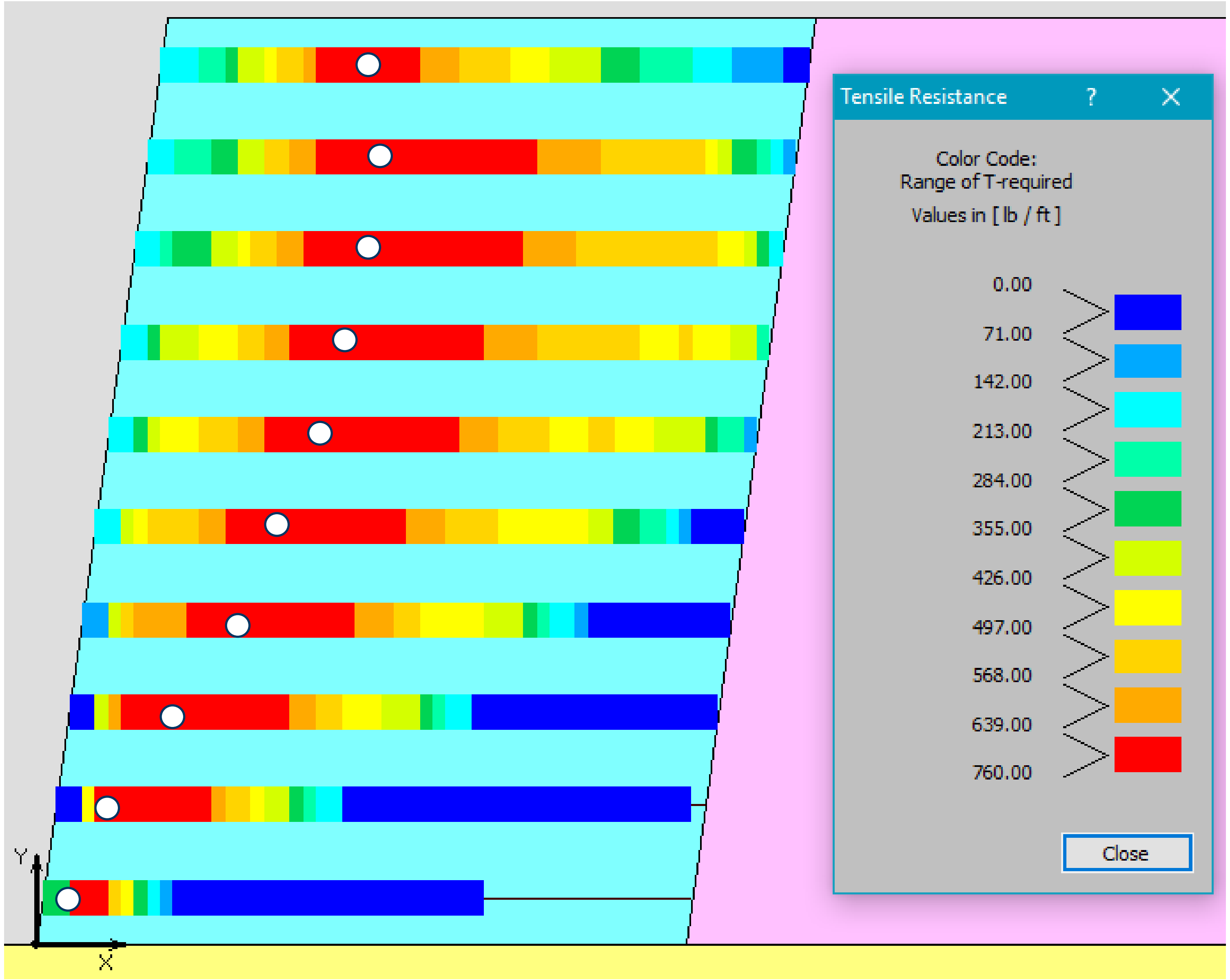
Inverse of Safety Map…

- **Safety Map** finds the ***spatial*** distribution of the ***safety factors***, SF, in a reinforced soil mass

- Conversely, Internal Stability analysis in LE produces the tensile resistance needed for **Fs=SF=1.0 everywhere**

- The Internal Stability approach produces the ***baseline solution***: **Tension Map** means $T_{req}(x)$, including $\mathbf{T_{max}}$ and $\mathbf{T_o}$ for each layer ➔ It leads to a rational and robust selection of reinforcement strength and facing

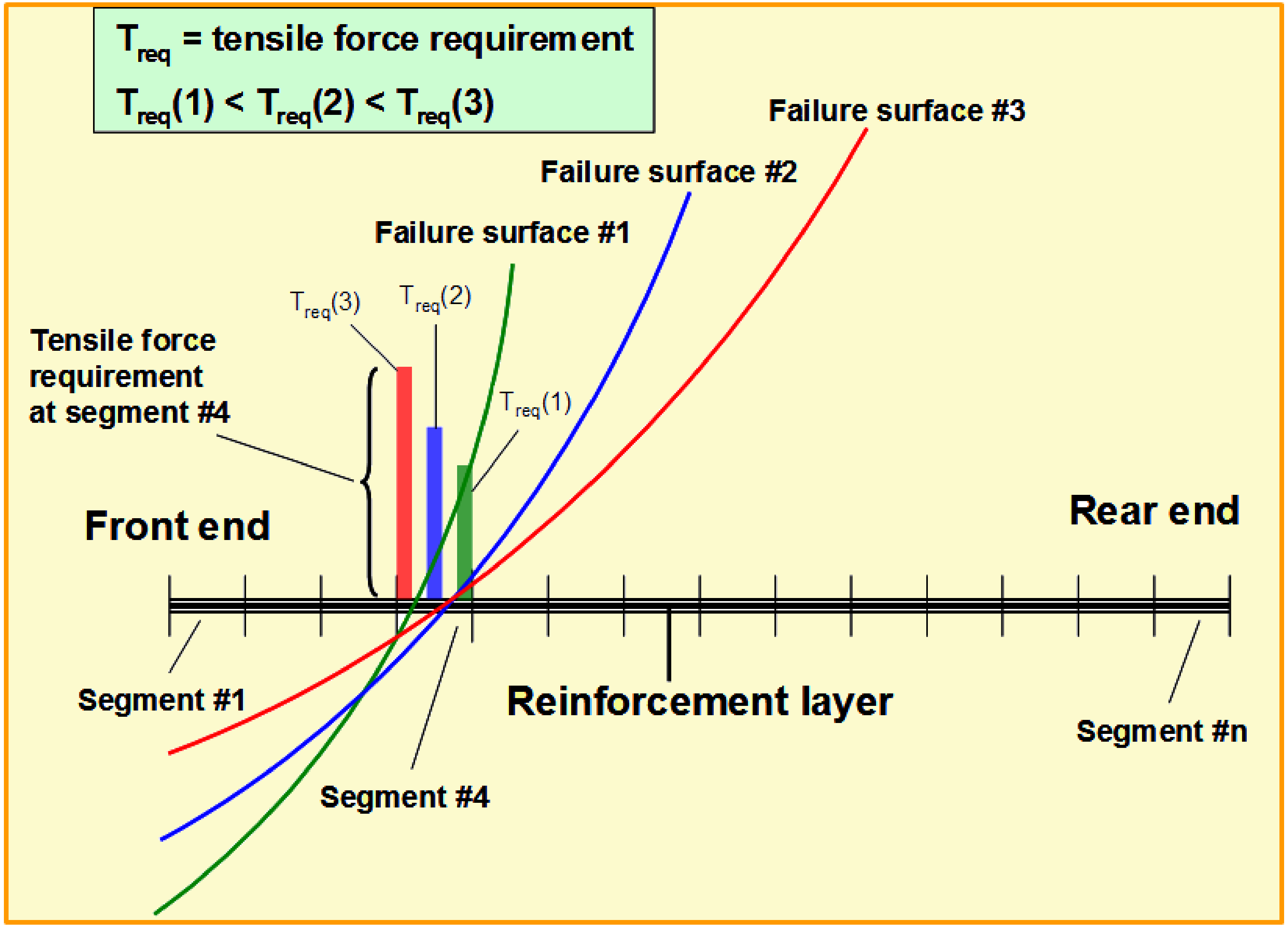
Tension Map: Visualization of $T_{req}(x)$



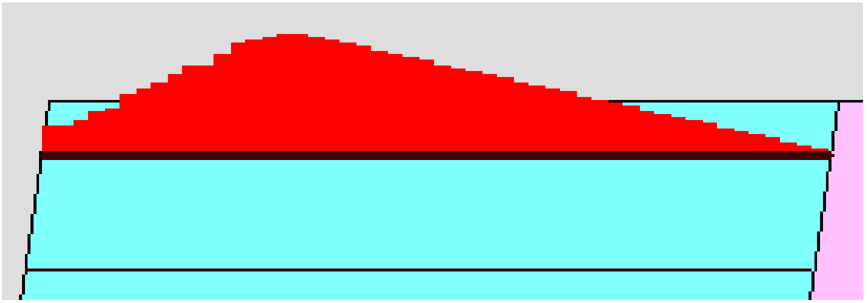
The Framework: Process in Nutshell

- Check numerous test bodies setting SF=1.0 and calculating $T_{req}(x)$ for each layer ➔ Use a systematic top-down process
- For $T_{req}(x)$ distribution, likelihood of failure along any surface is same ➔ $T_{req}(x)$ therefore is termed **Baseline Solution ➔ Tension Map**
- The tension, $T_{req}(x)$, is limited by pullout at the rear and/or front ends
- $T_{req}(x)$ is the resistance needed locally to yield a structure at a limiting equilibrium state

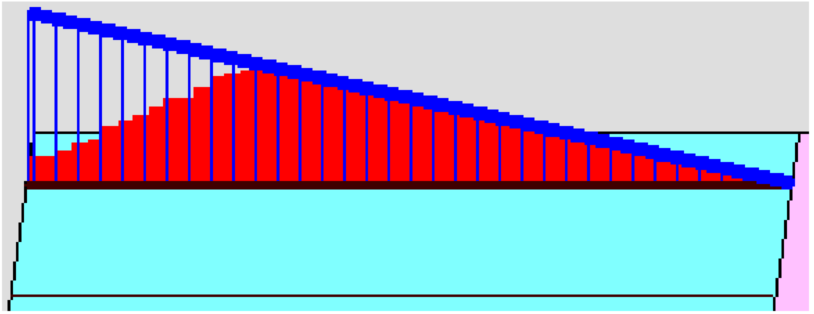
Maximization Update...



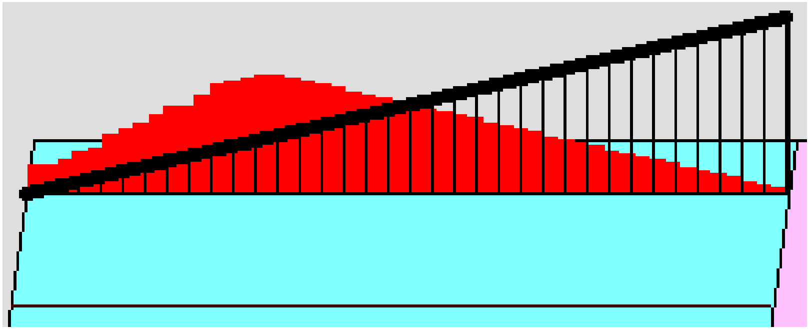
Details: Baseline & Pullout



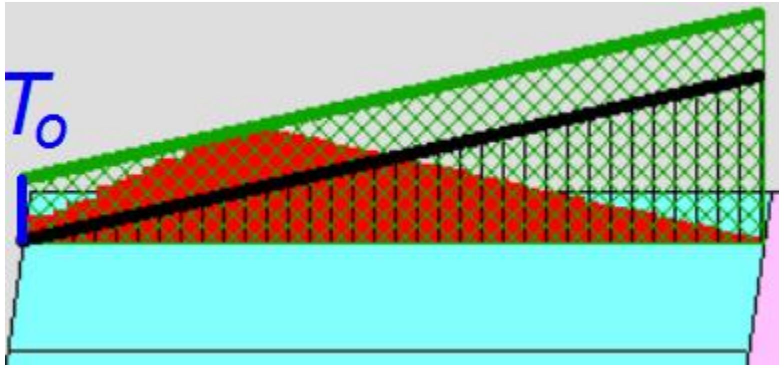
1. $T_{req}(x)$



2. Rear pullout constraint



3. Front pullout… oops



4. Adjust front pullout
➔ Upwards shift is T_o

Geosynthetic Reinforced Wall (Alabama, Photo: Feb 2007)

Percolating water ➔ Decrease in σ_v' ➔ Decrease in front pullout resistance ➔ Geogrid may not mobilize the needed resistance thus relying on added resistance from connections ➔ Connections strength for upper layers exceeded ➔ Failure

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Advancement of Current Design

- Apply the LE design approach in two stages: ***Internal Stability*** and ***Global Stability***
- **Stage 1: Internal stability** - Find $T_{req}(x)$ in all reinforcements - Baseline Solution
- Consider geometry, loading conditions, reinforcement layout, pullout resistance, any batter, water, seismicity, etc.
- **Stage 2: Global stability** - consistent with current design ➔ Standard slope stability analysis

Stage 1: Internal Stability

- Find $T_{req}(x)$ including T_{max} & T_o (connection)
- Determine $\max(T_{max})$ to select geosynthetic
- $LTDS = F_{s\text{-}strength} \times \max(T_{max})$ where $F_{s\text{-}strength} = 1.5$ [**Note:** In LRFD **LF**≥1.35 and **RF**≥0.9 ➔Hence $F_{s\text{-}strength}$ needs to be > (1.35/0.9)=1.50…]
- $T_{ult} = LTDS \times RF_{cr} \times RF_d \times RF_{id}$

- **Stage 1** is a rational and robust alternative to existing approaches➔ Ensures that there is no overstressing of reinforcement

Stage 2: Global Stability

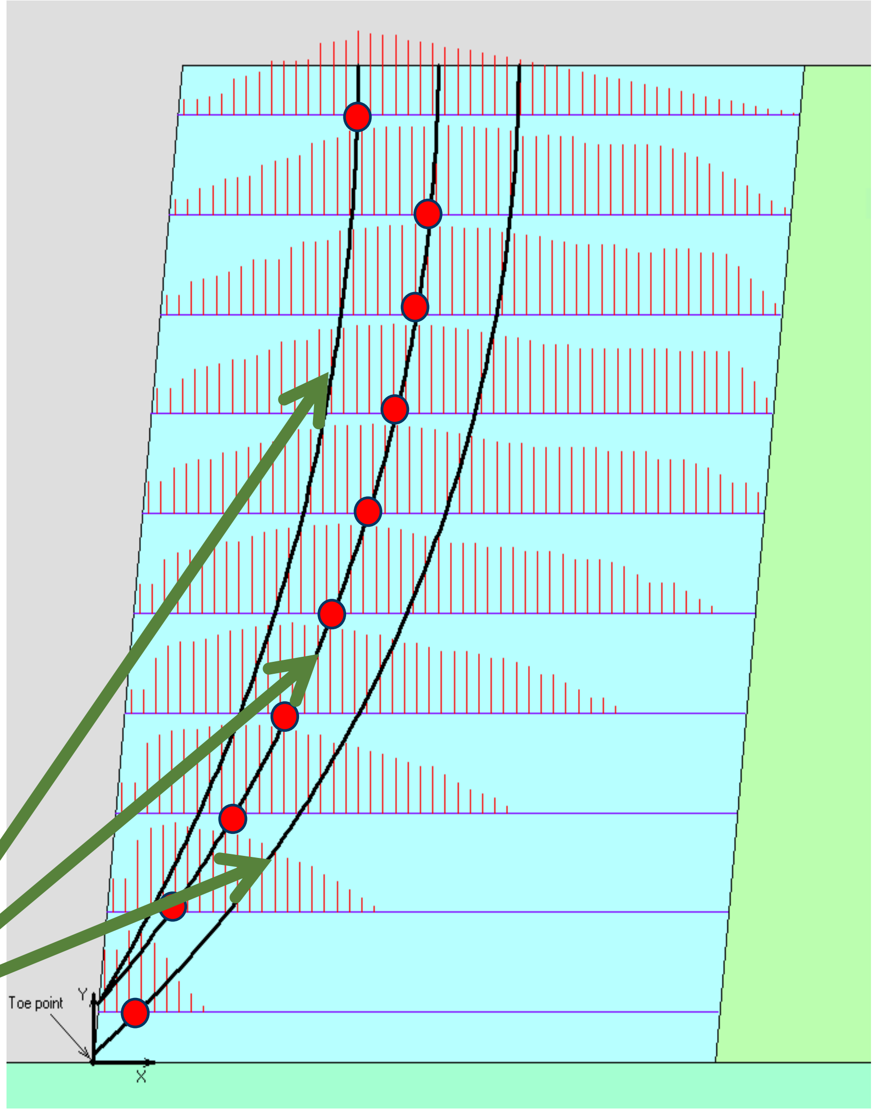
- Select reinforcement and facing following **Stage 1**
- Conduct global slope stability analysis to ascertain that for the selected facing, layout and strength of reinforcement, $F_s \geq 1.30$ for all feasible failure geometries
- Increase the length and/or strength of reinforcement, if needed, to meet the prescribed on soil strength Fs

Stage 2 Conducts Global Stability
Why use then Internal Stability?

- Reinforcement resistance in Global Stability is evenly divided amongst all layers ➔ May results in T_{max} that is smaller than in Internal Stability ➔ Global ignores local demand through ‘smearing’ or averaging

*- Global Stability tells us **nothing** about **connection** load, T_o*

Global Stability: Locus of T_{max} is NOT on a singular surface.



So Why Stage 2 is important too?

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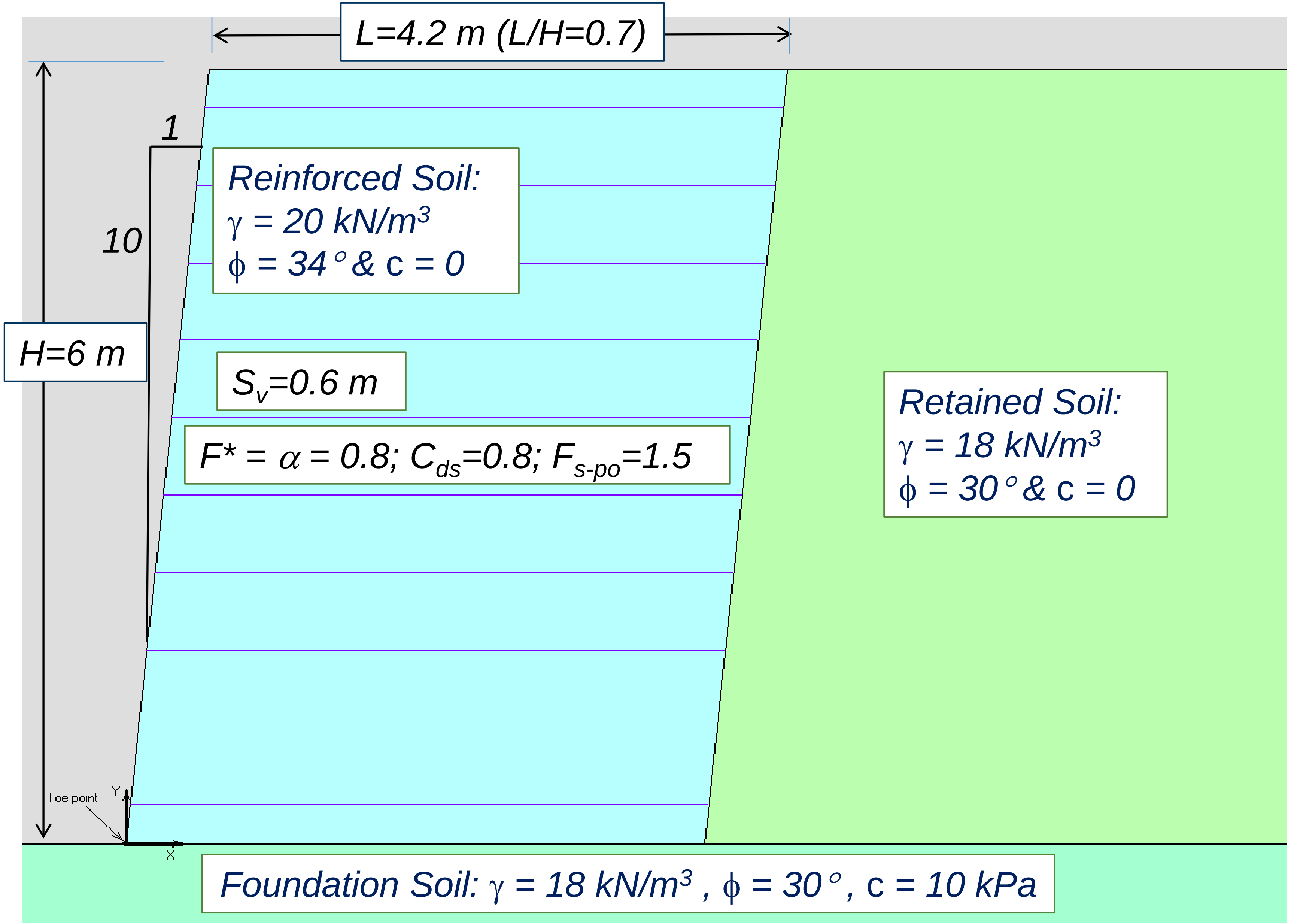
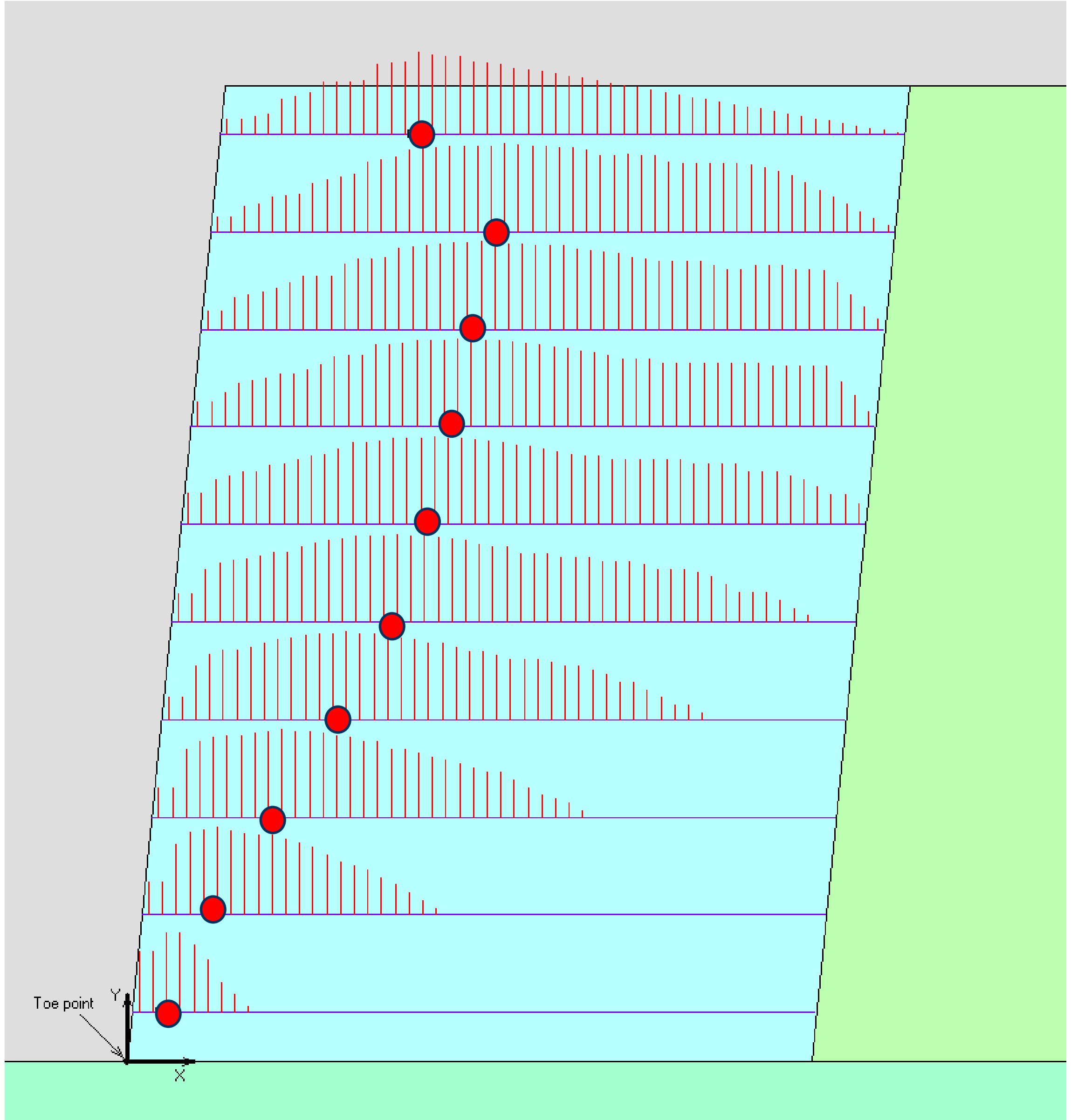
Benchmark Problem

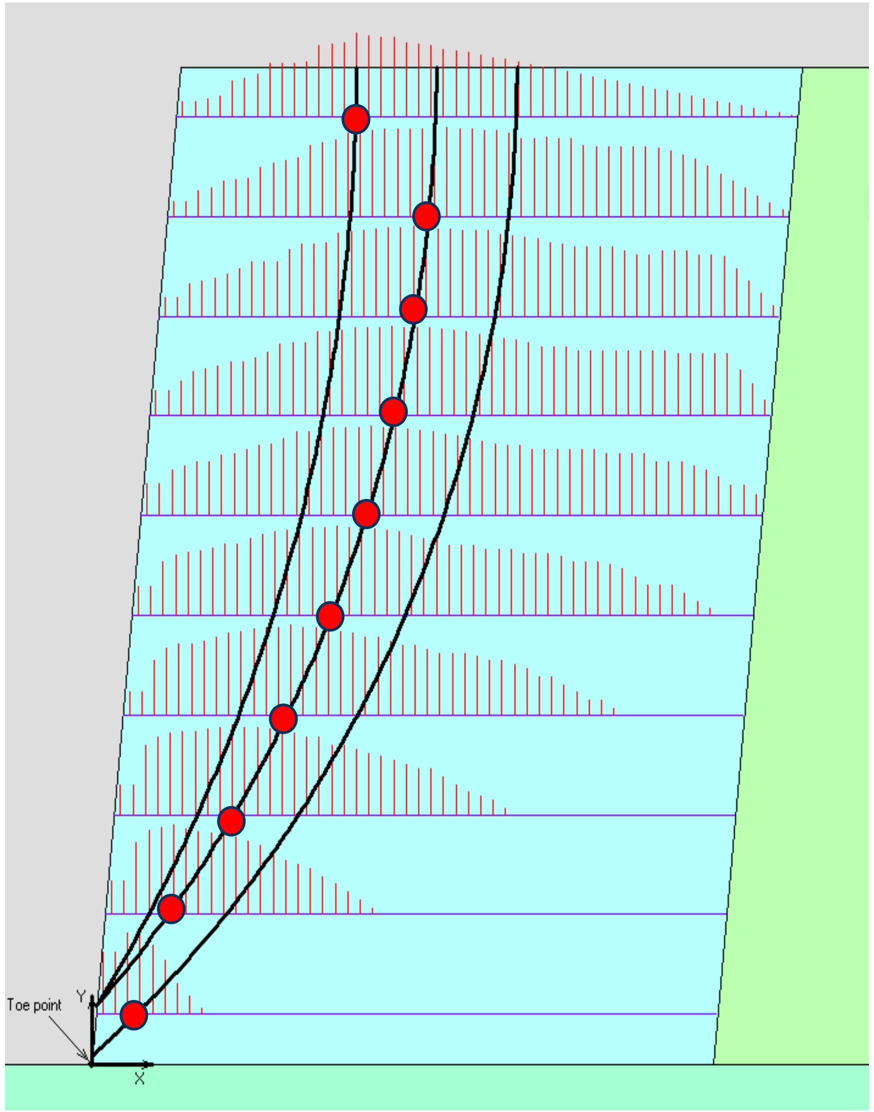


Computed Distribution of $T_{req}(x)$



Computing T_{max} in Internal Stability: Critical Circles

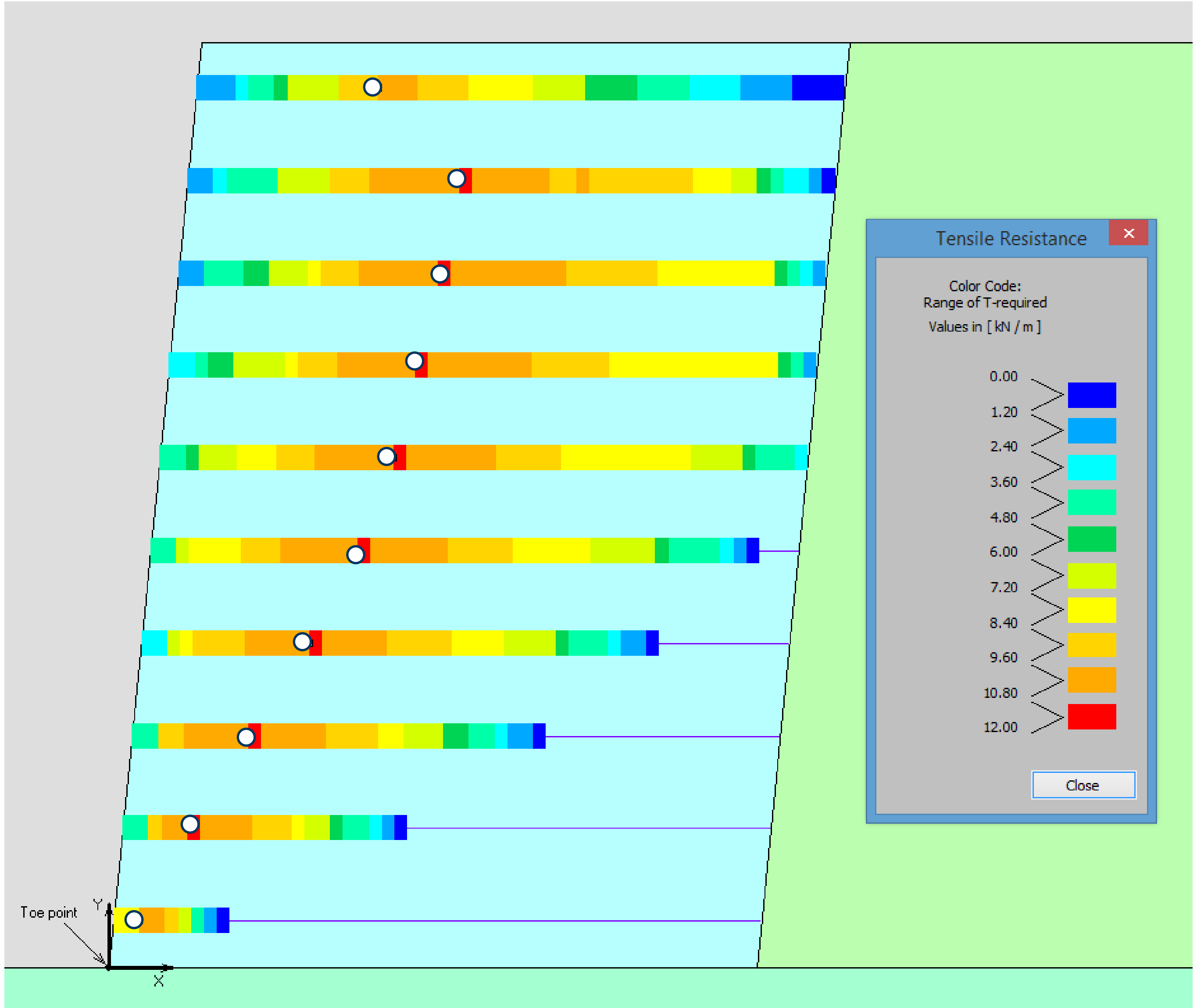
1. *Hypothesis in AASHTO: Locus of T_{max} is defined by a singular slip surface.*
   Is it?
2. *Well-defined active and resistant zones.*
   Is it?



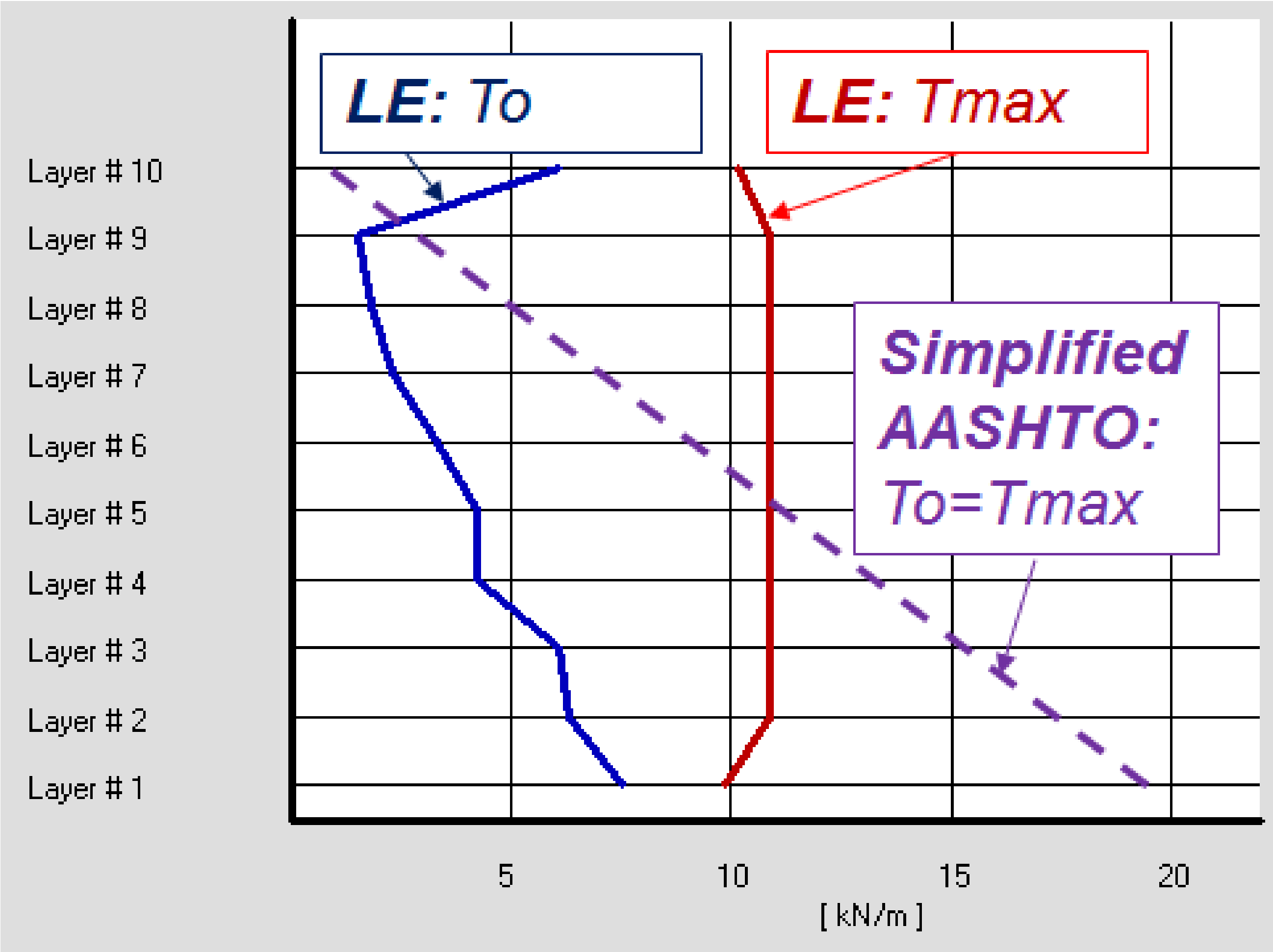
Tension Map

The mobilization of tension in each reinforcement can be visualized through the Tension Map →

Note location of T_{max}



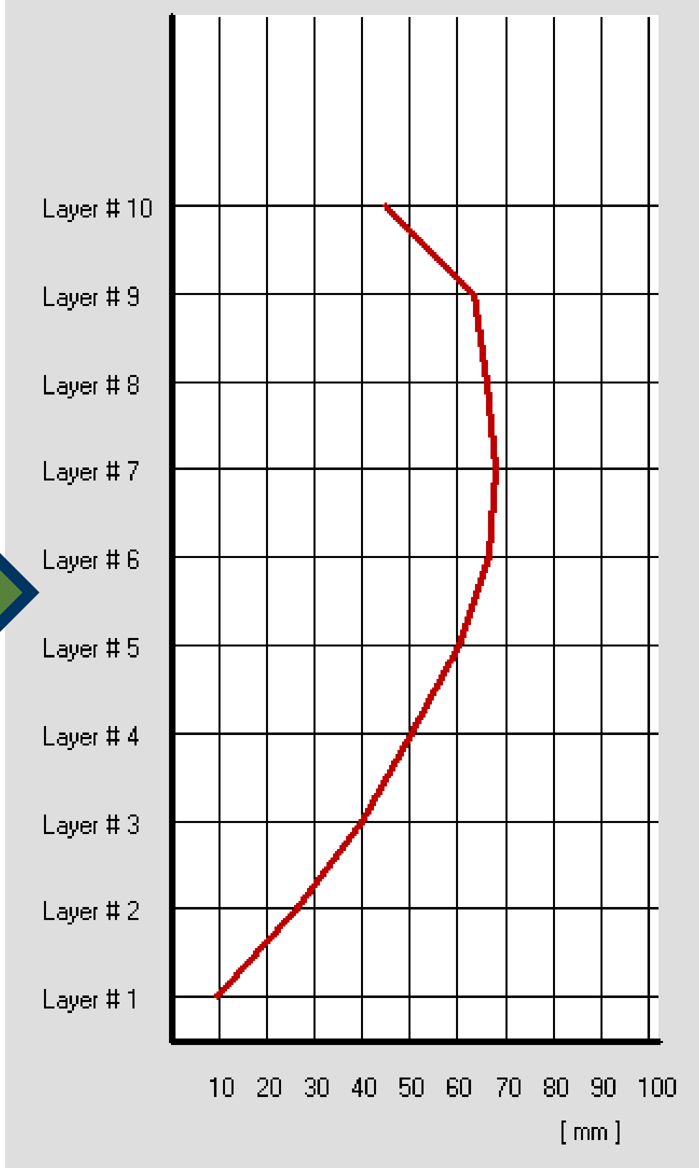
T_{max} and T_o Distribution



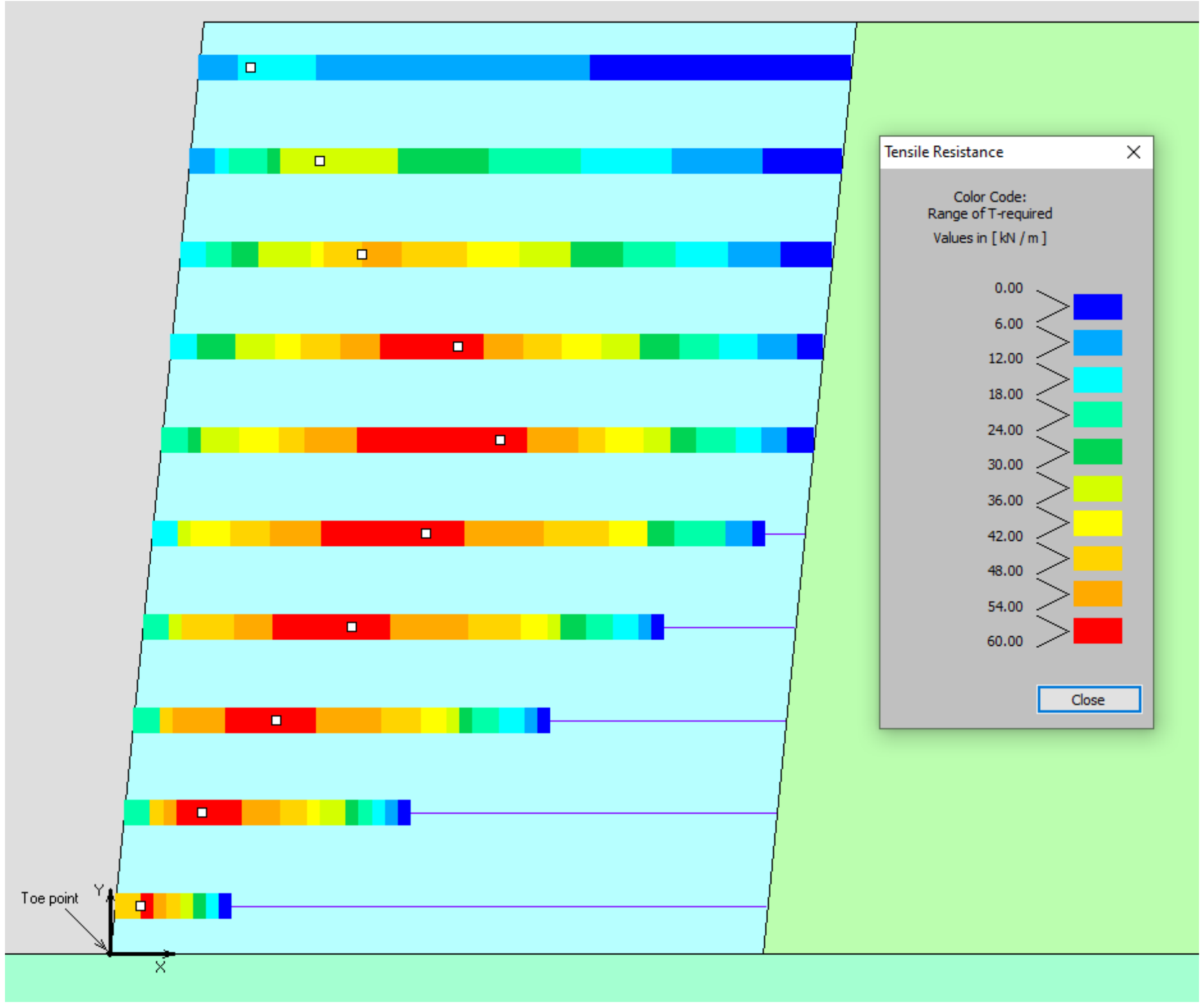
max(T_{max}): **LE** ➔ 10.9 kN/m **AASHTO** ➔19.3 kN/m

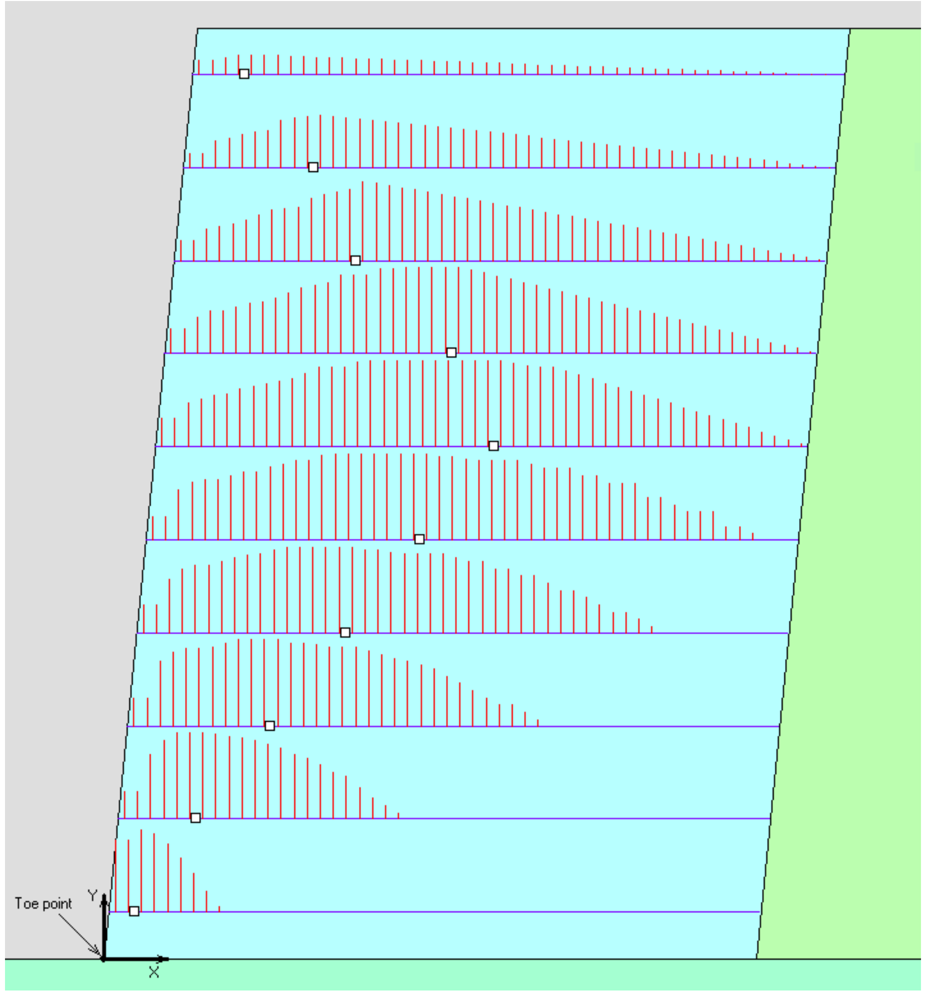
Horizontal Displacement Distribution

$T_{req}(x)$ for Fs=1.0 allows for ***Estimation*** *of the lateral displacement at a limit state e.g., for J=500 kN/m*

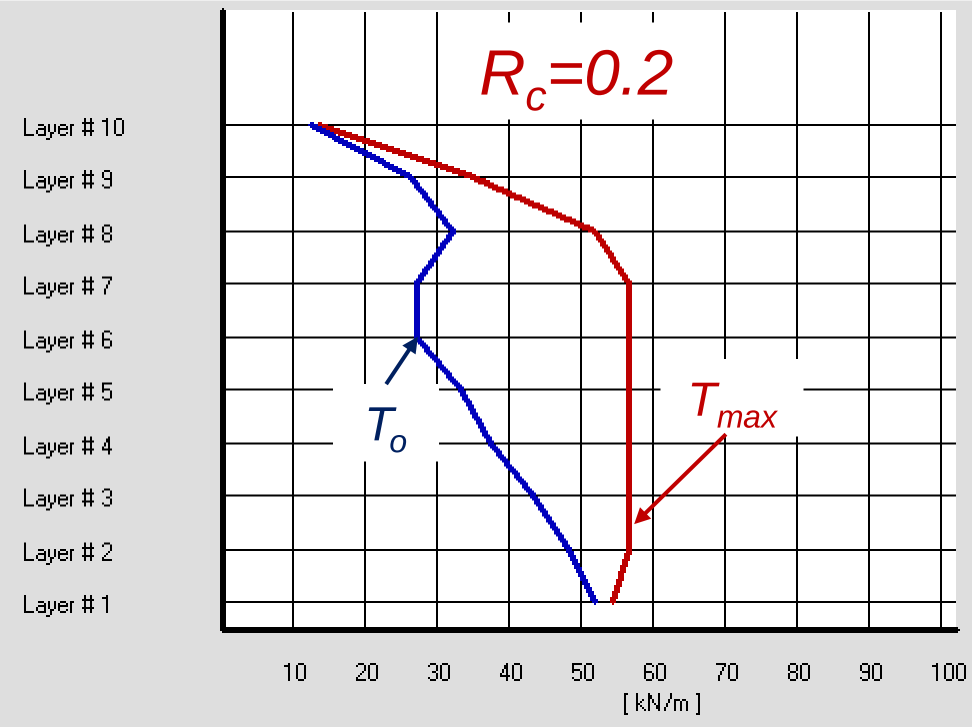


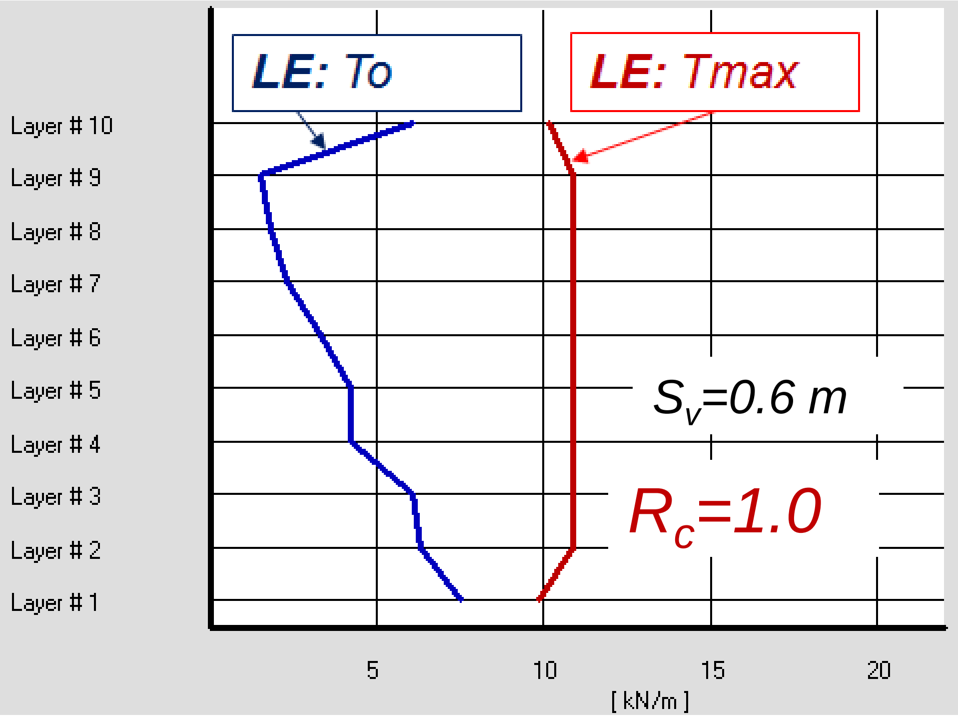
Computed Distribution of $T_{req}(x)$: $R_c=0.2$





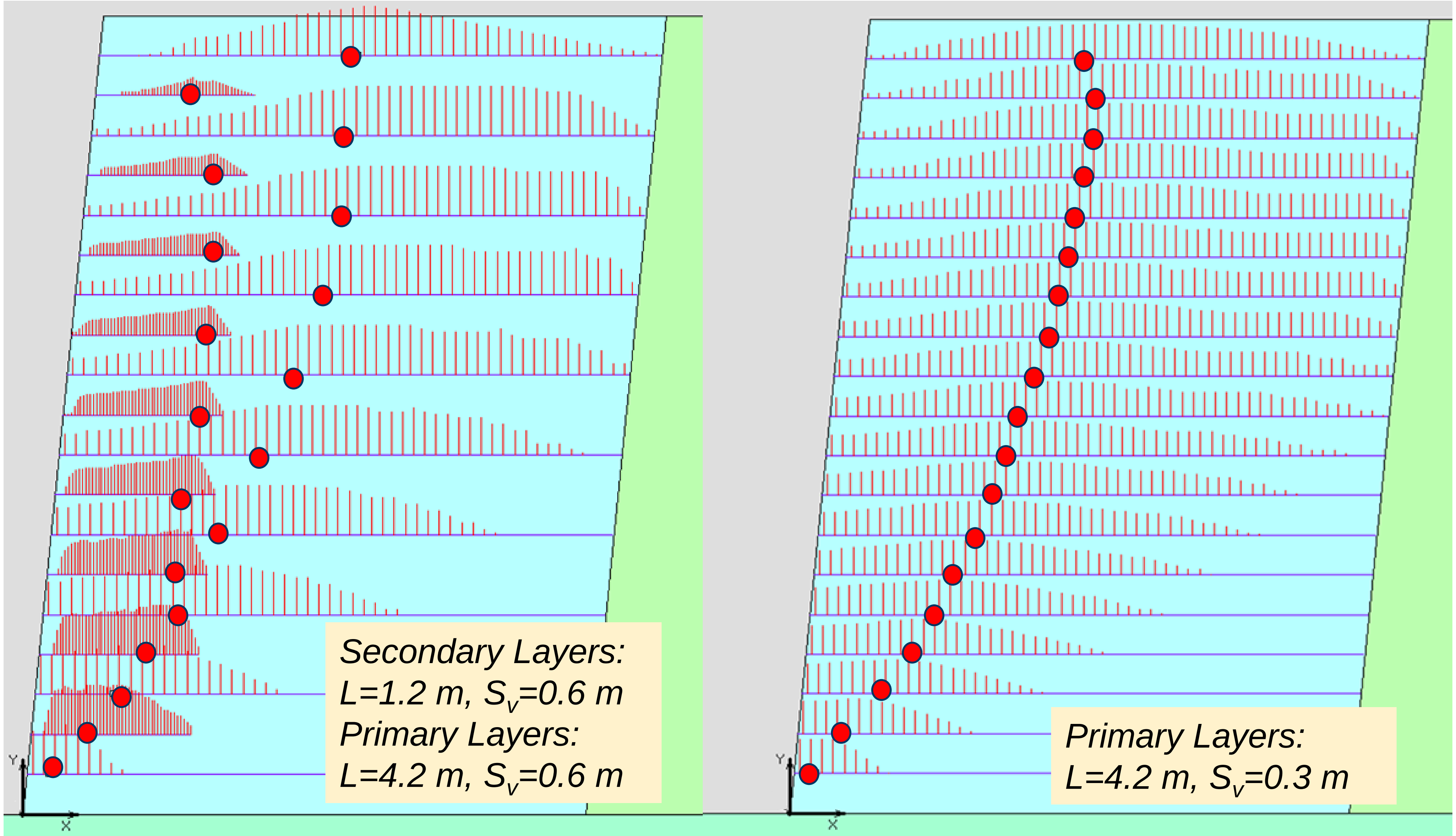
Effects of R_c on T_{max} and T_o



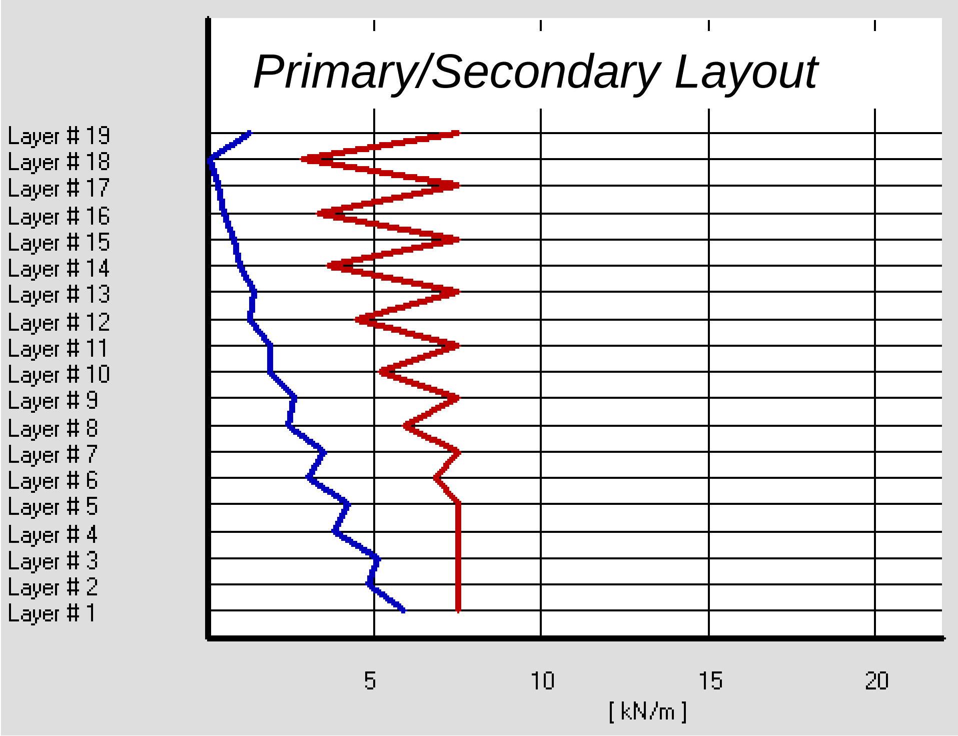


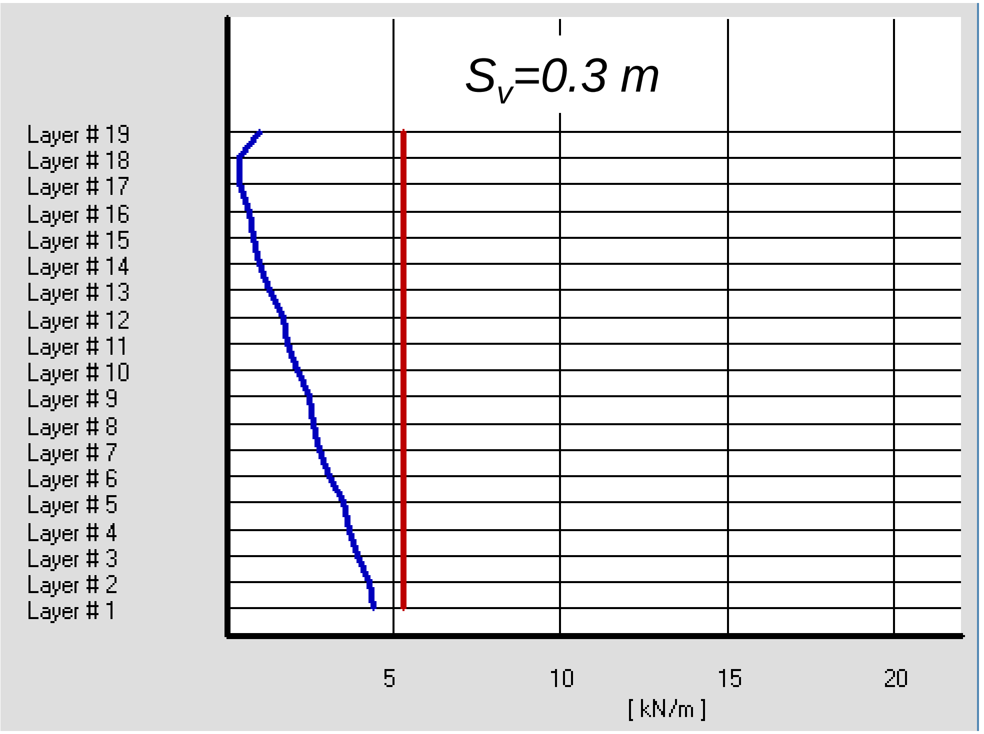
Note the different drawing scale for T_{max} and T_o

Effects of Secondary Layers



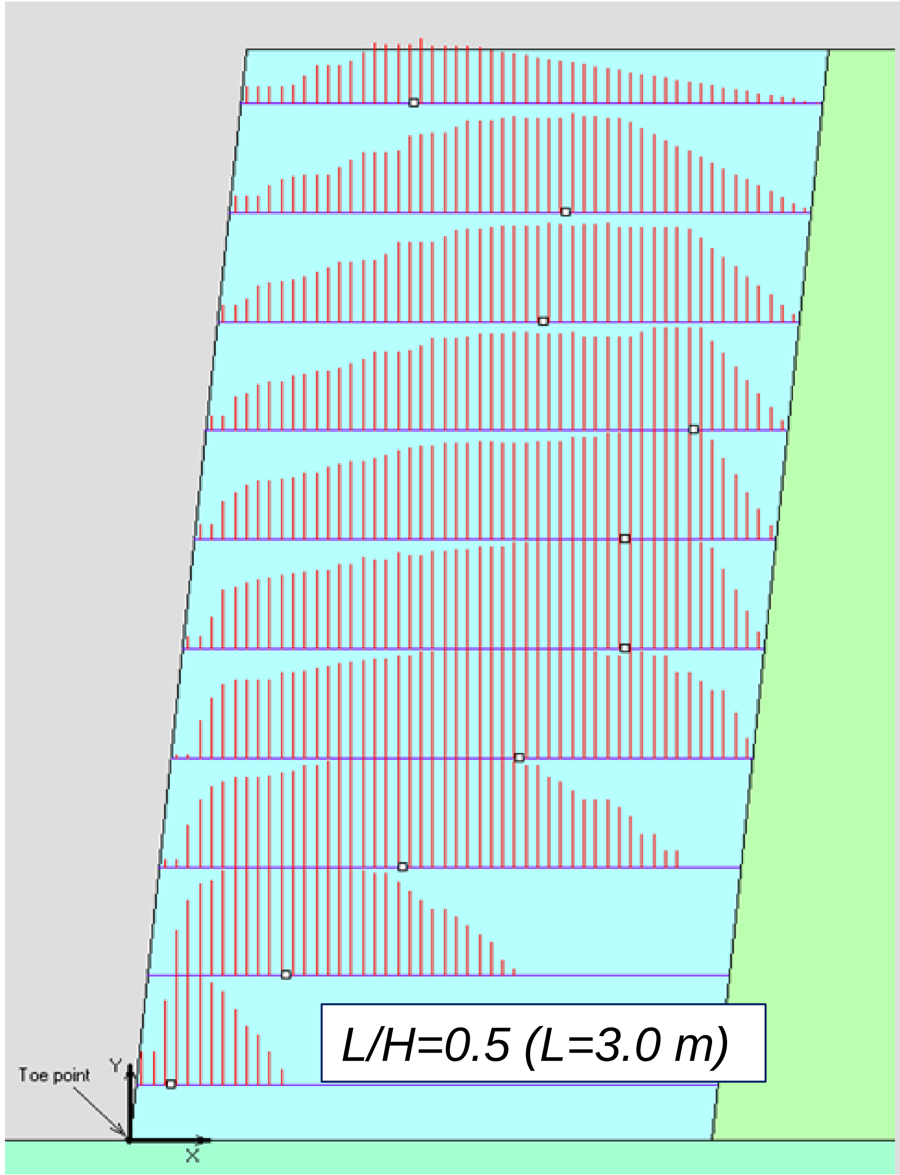
T_{max} and T_o: Secondary versus Close Spacing

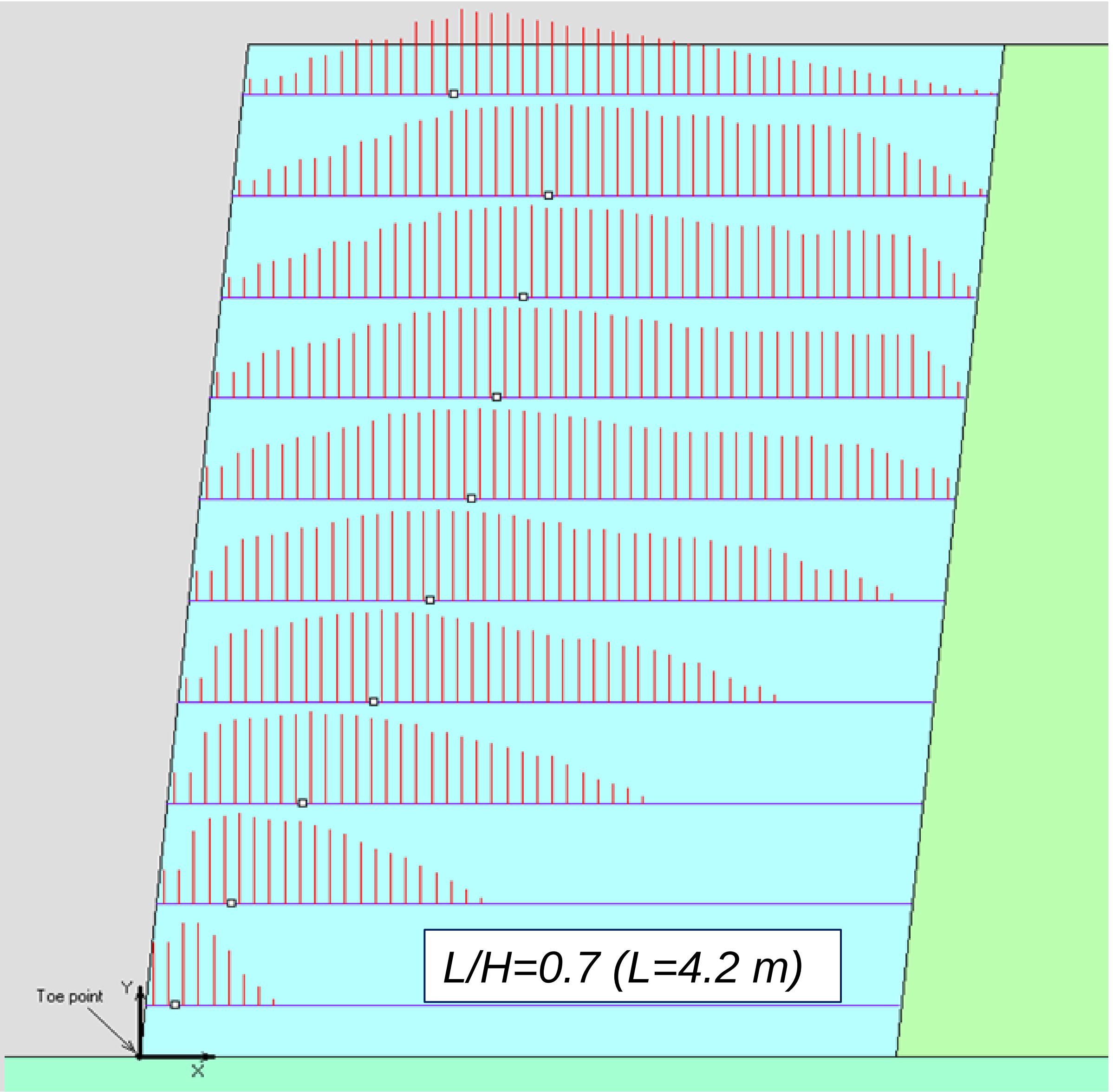




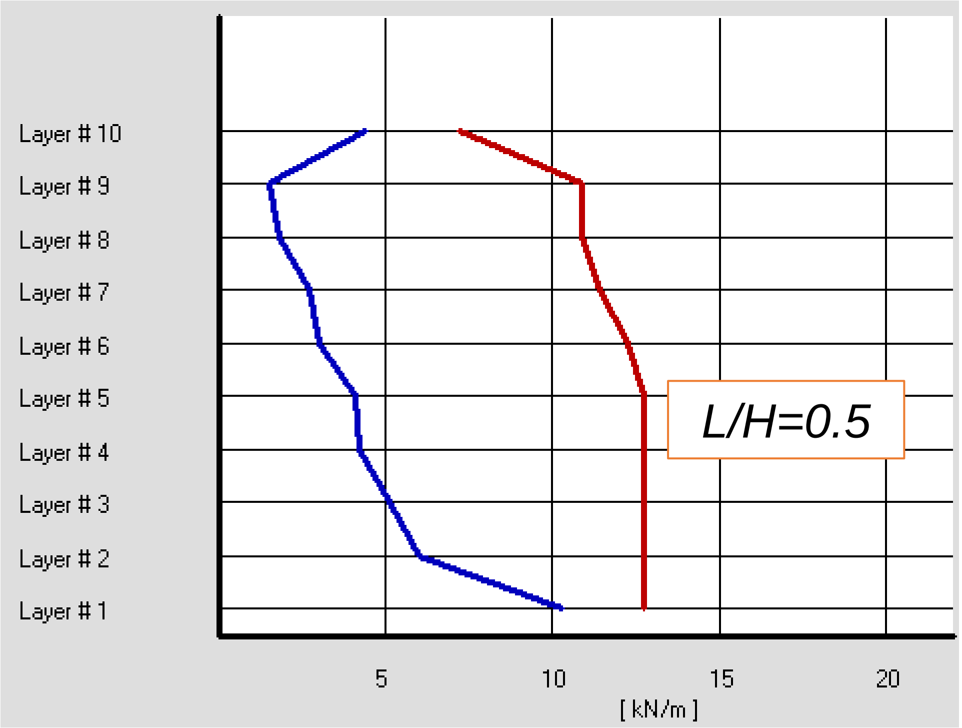
Depending on relative length of secondary reinforcement, T_{max} may decrease. Generally it has significant effects on T_o (connection loads).

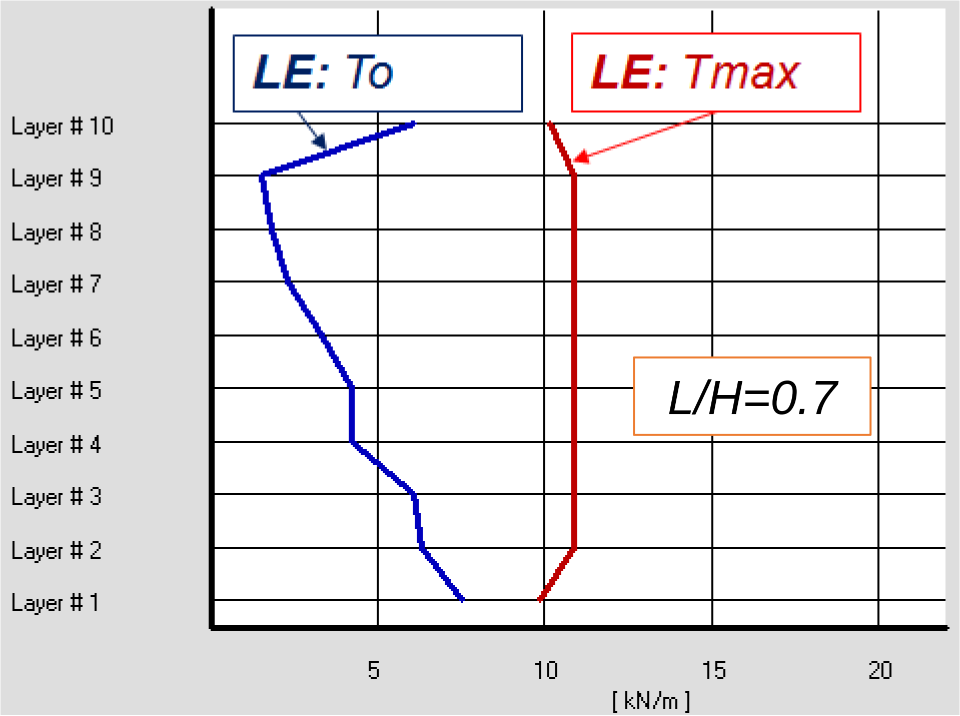
Effects of Shorter Reinforcement





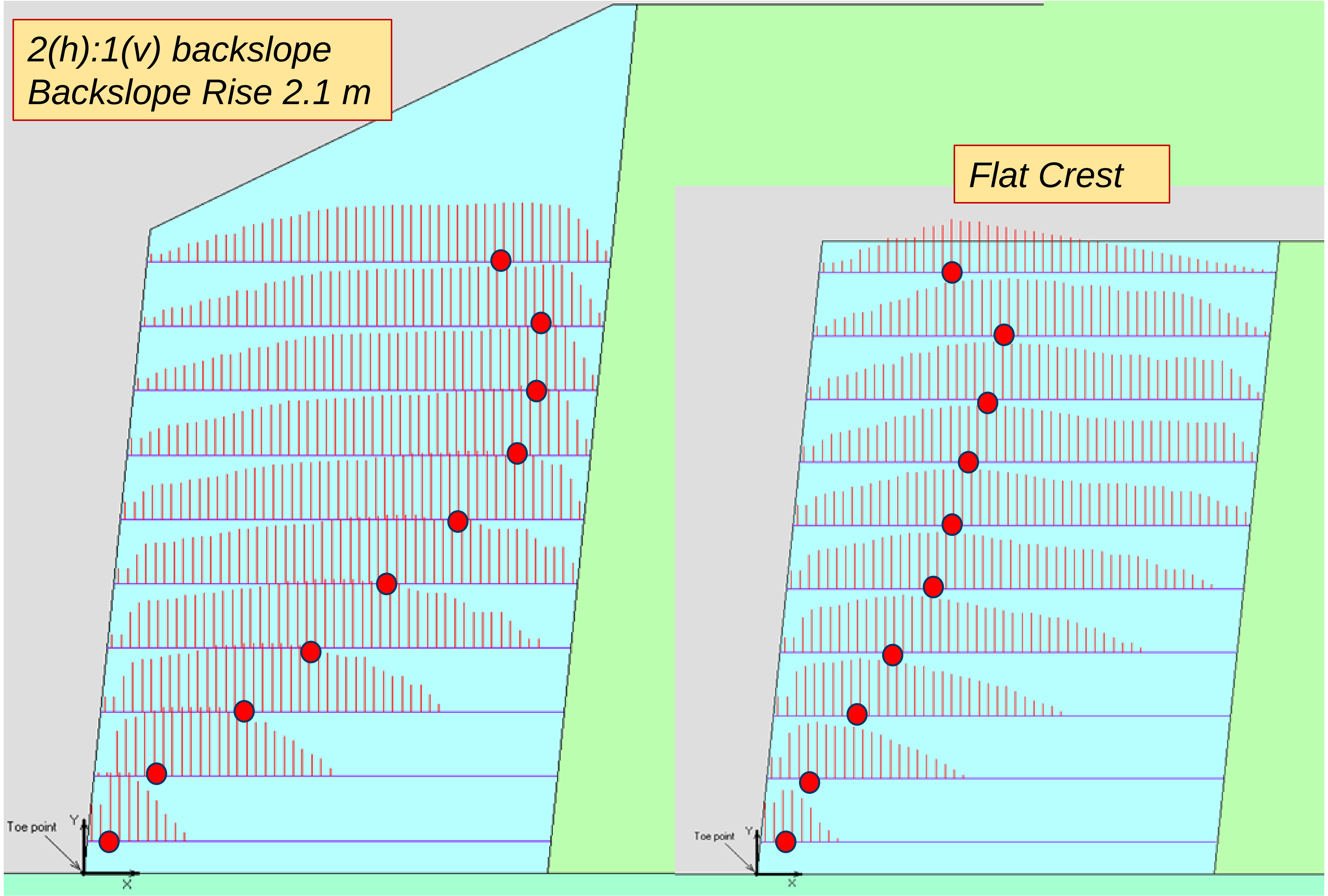
Effects of Shorter Reinforcement: T_{max} and T_o



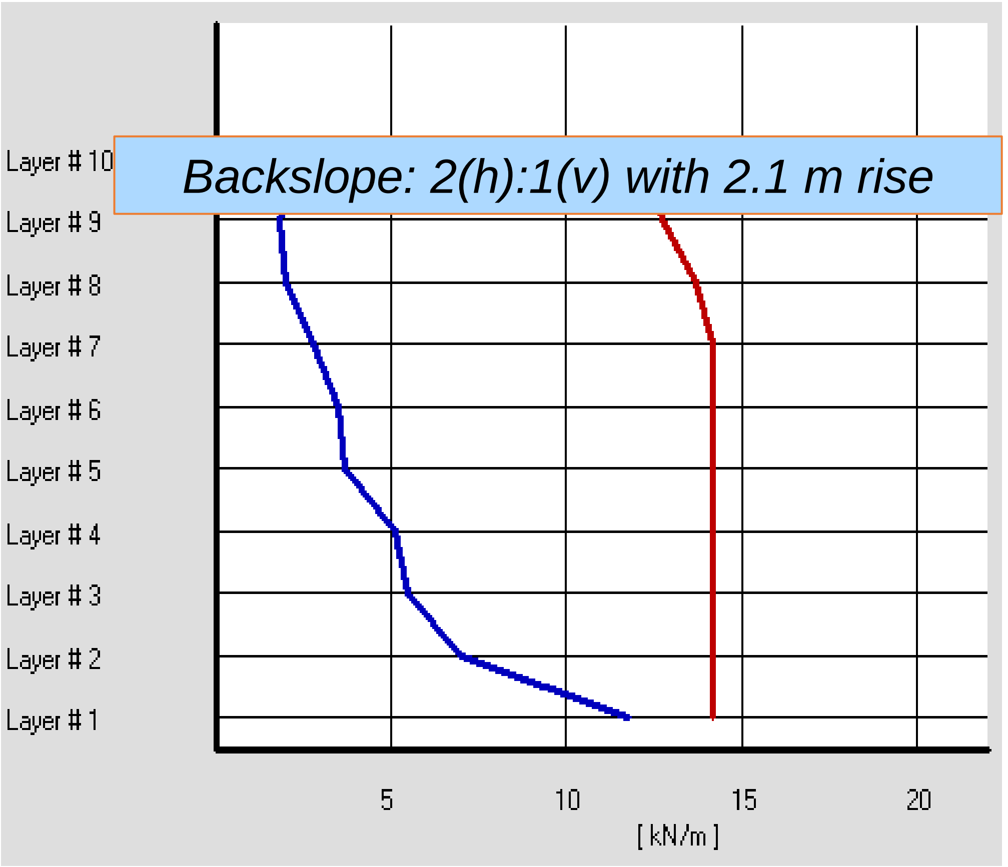


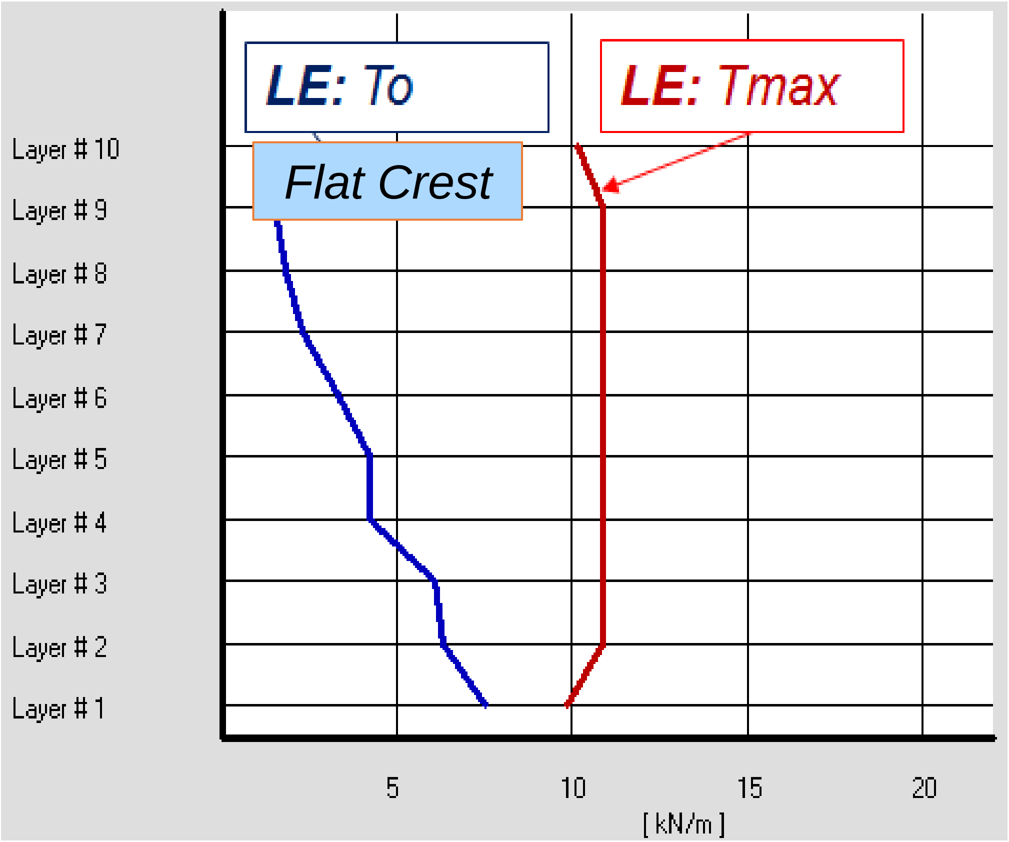
Generally, lower layers carry higher load due to compound failures ➔ Upper layers need to contribute less to produce Fs=1.0 ➔ Top layer carries less load thus resulting in smaller T_{max} and T_o

Effects of Backslope



Effects of Backslope: T_{max} and T_o

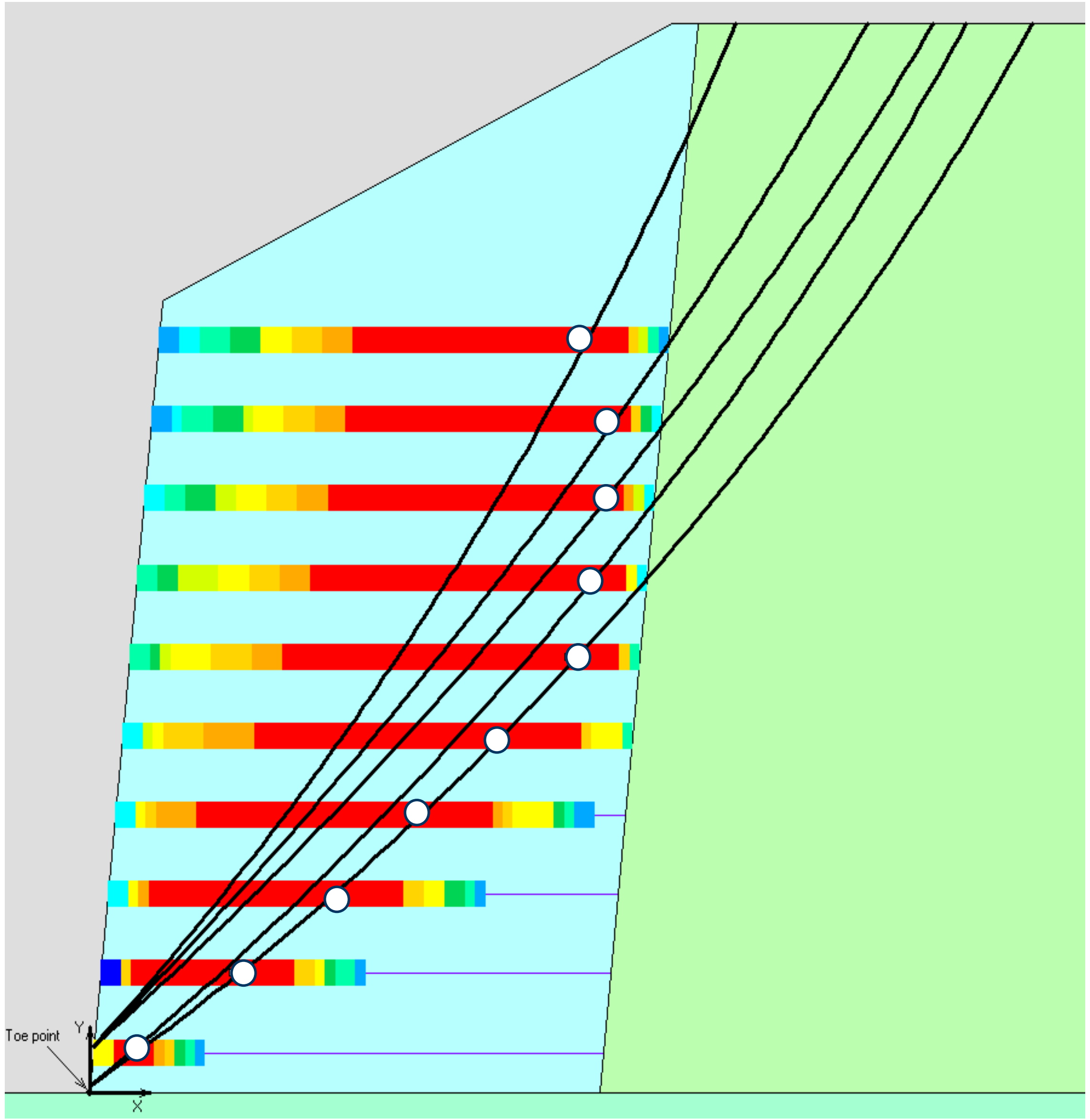




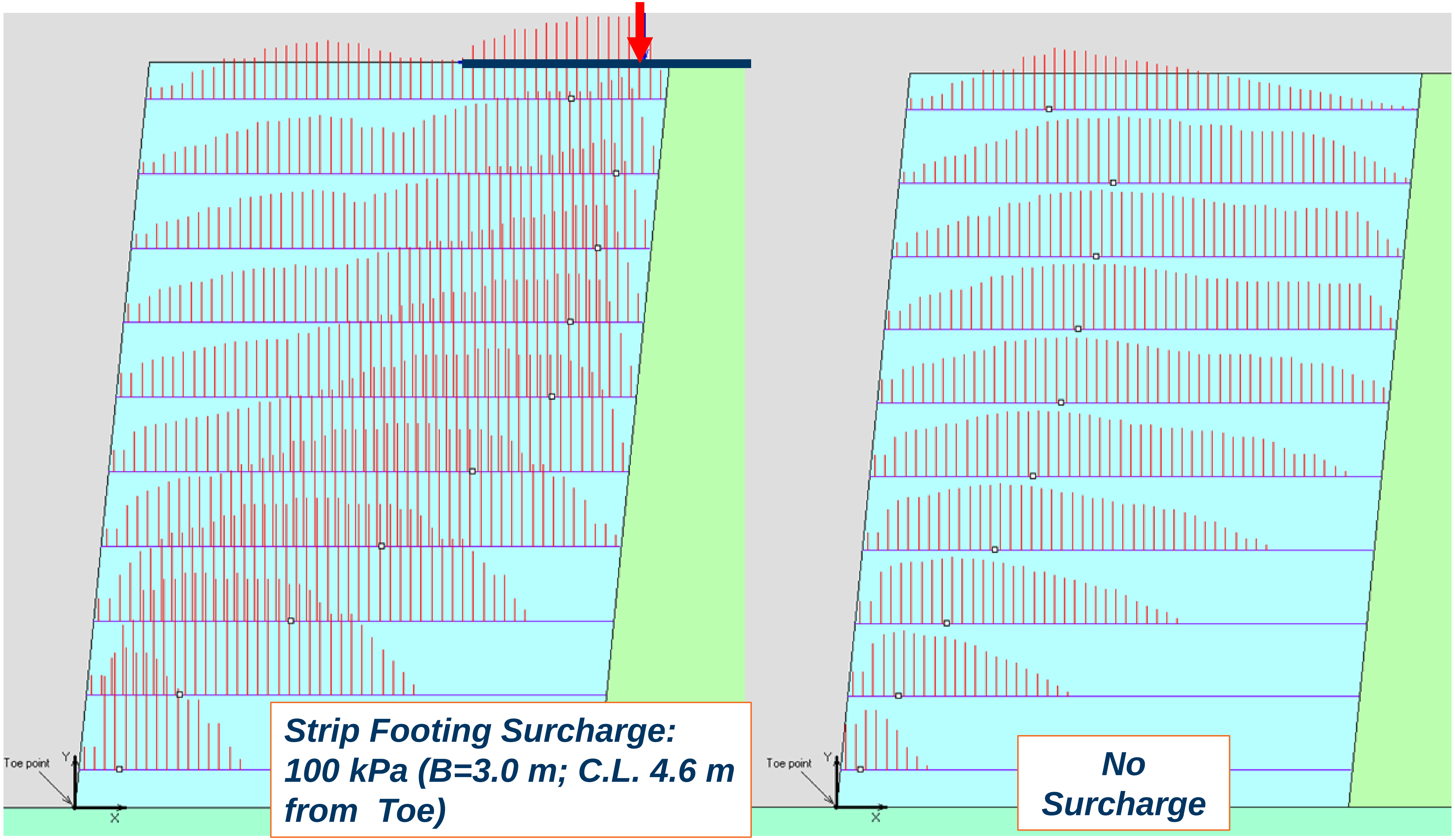
Computing T_{max} in Internal Stability: Critical Circles

Note:
Global Stability ➔ Top 4 layers are not needed for stability.

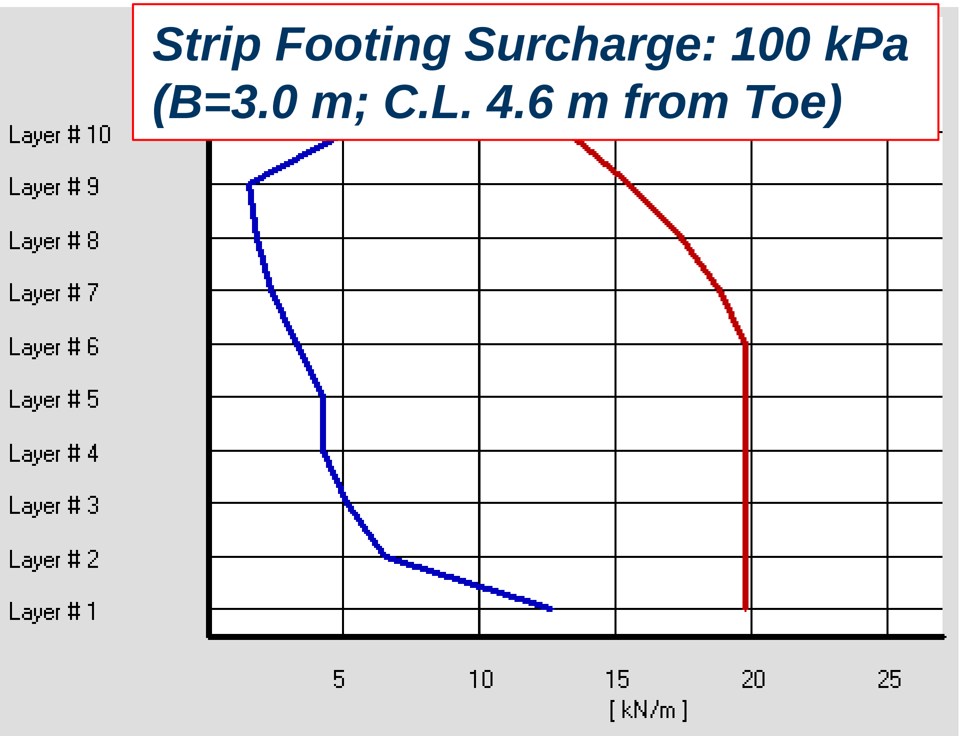
Baseline Solution,
Stage 1 ➔
Identifies the need for these layers!

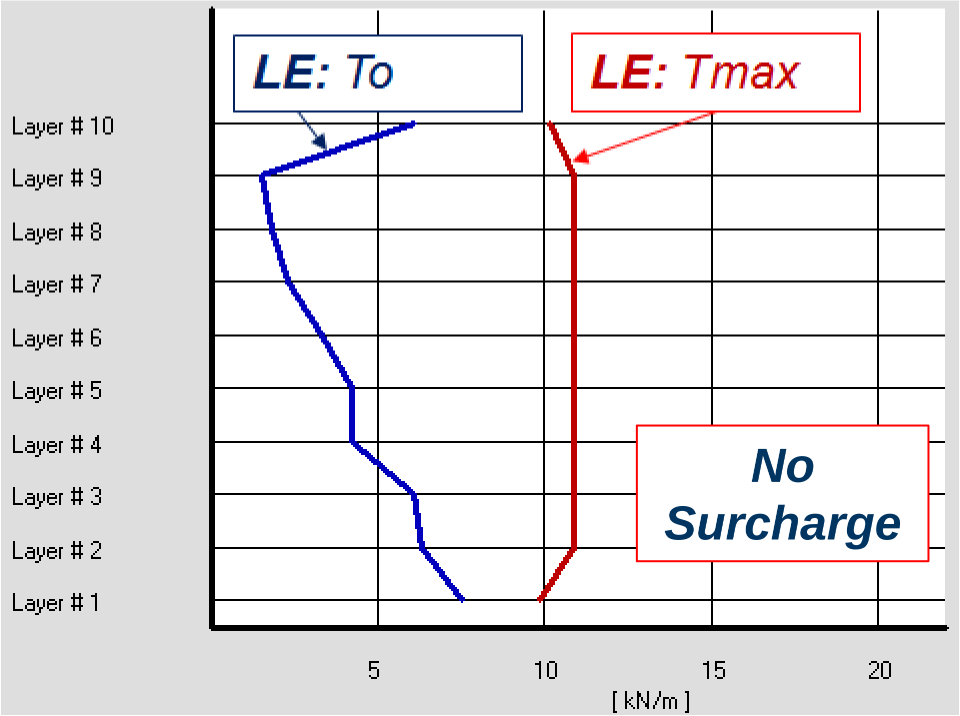


Effects of Surcharge (Dead Load)

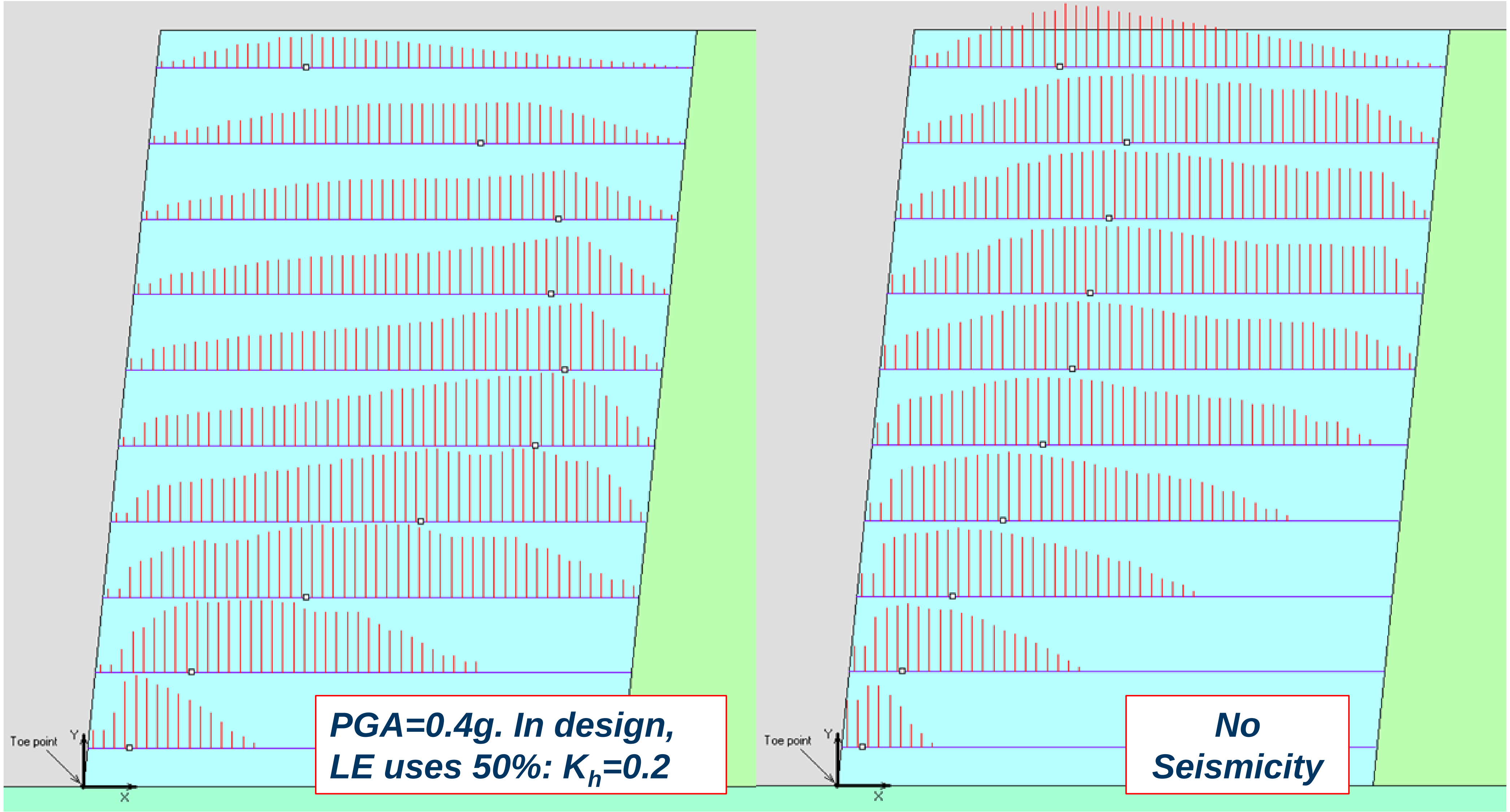


Effects of Surcharge: T_o and T_{max}

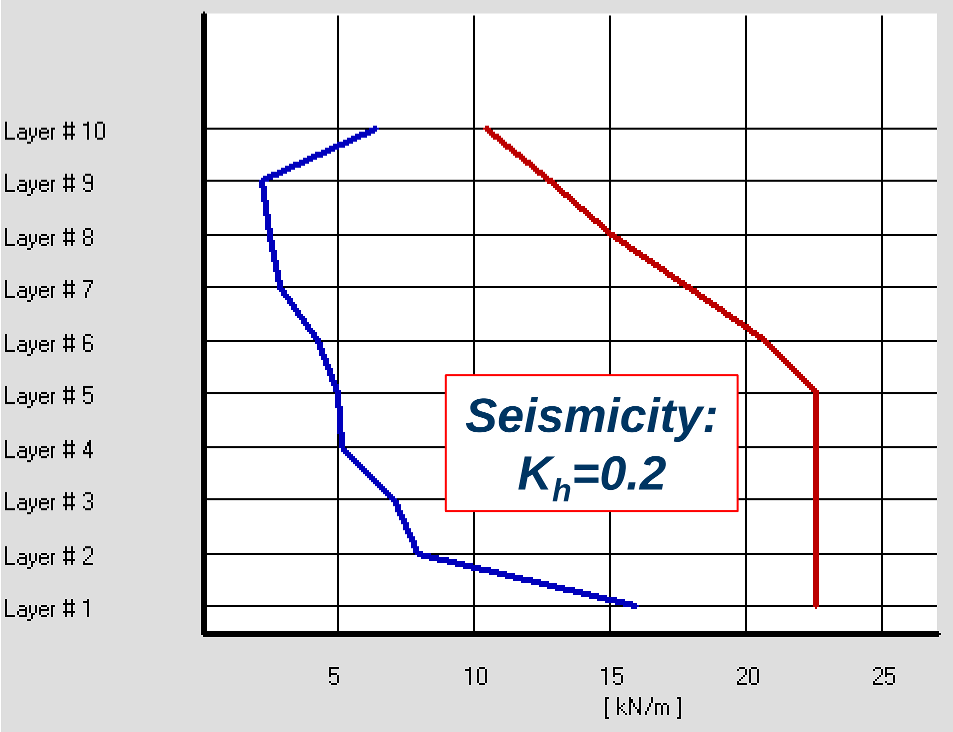


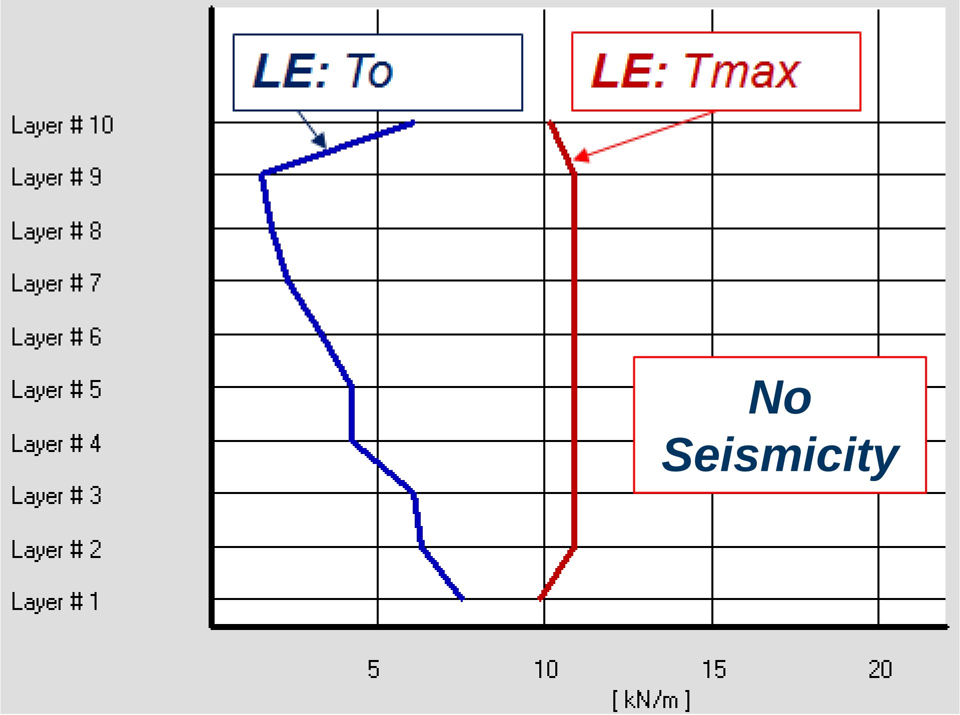


Effects of Seismicity

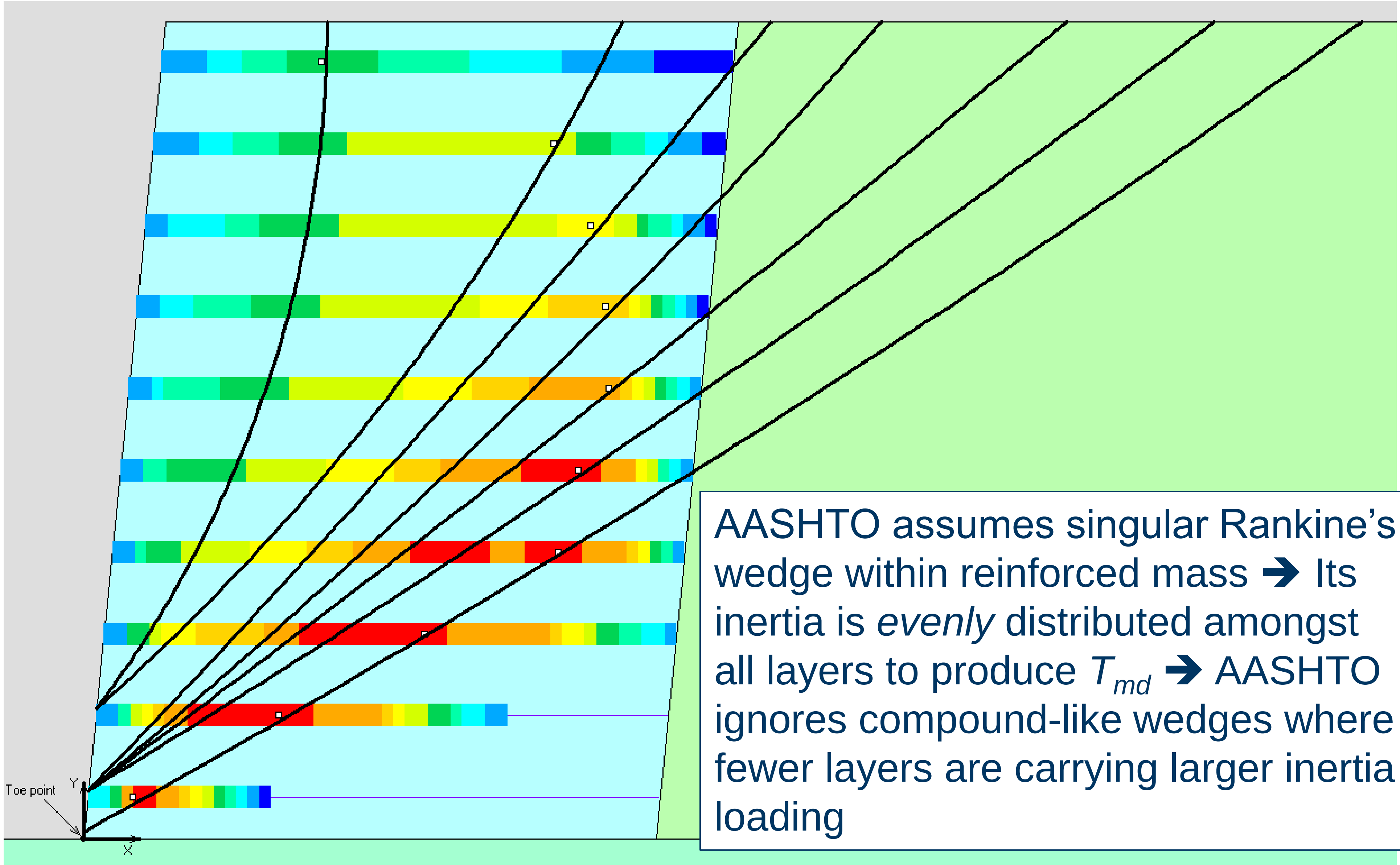


Seismic Effects: T_o and T_{max}



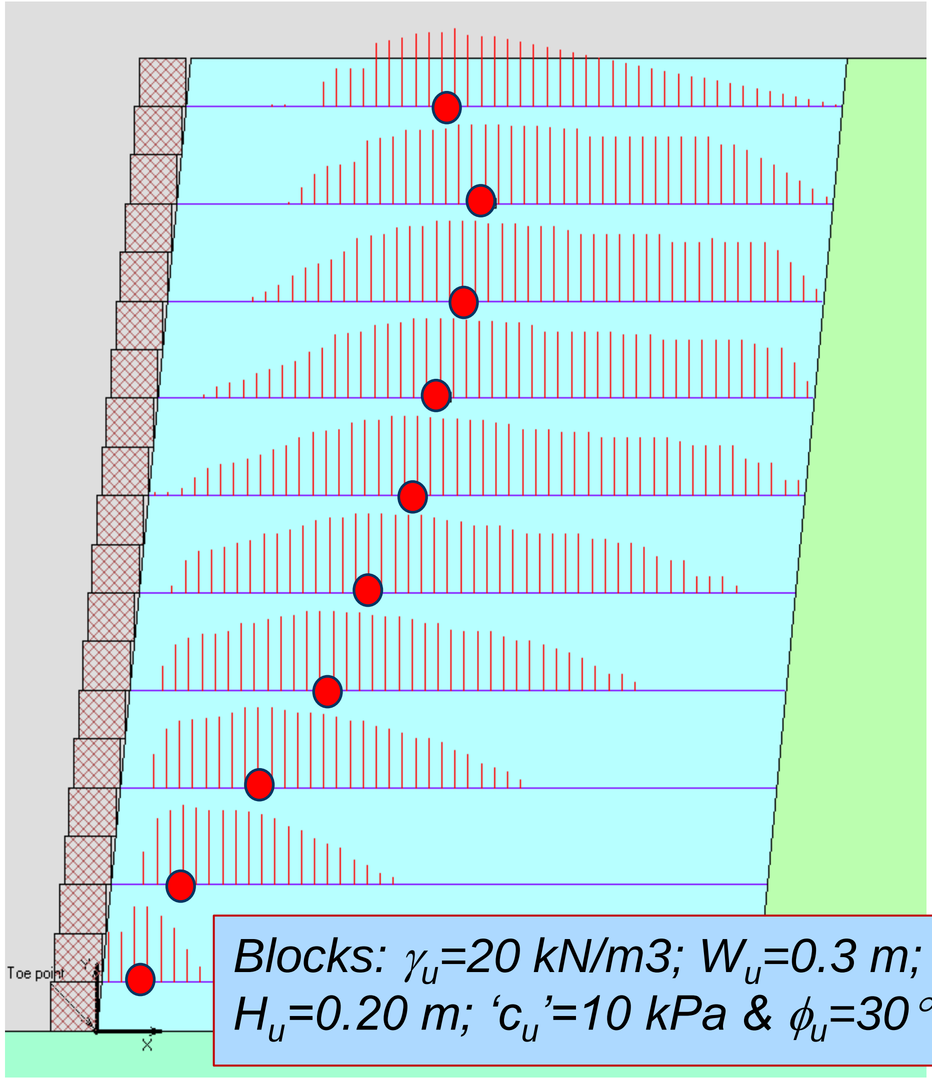


Critical Circles Rendering T_{max}

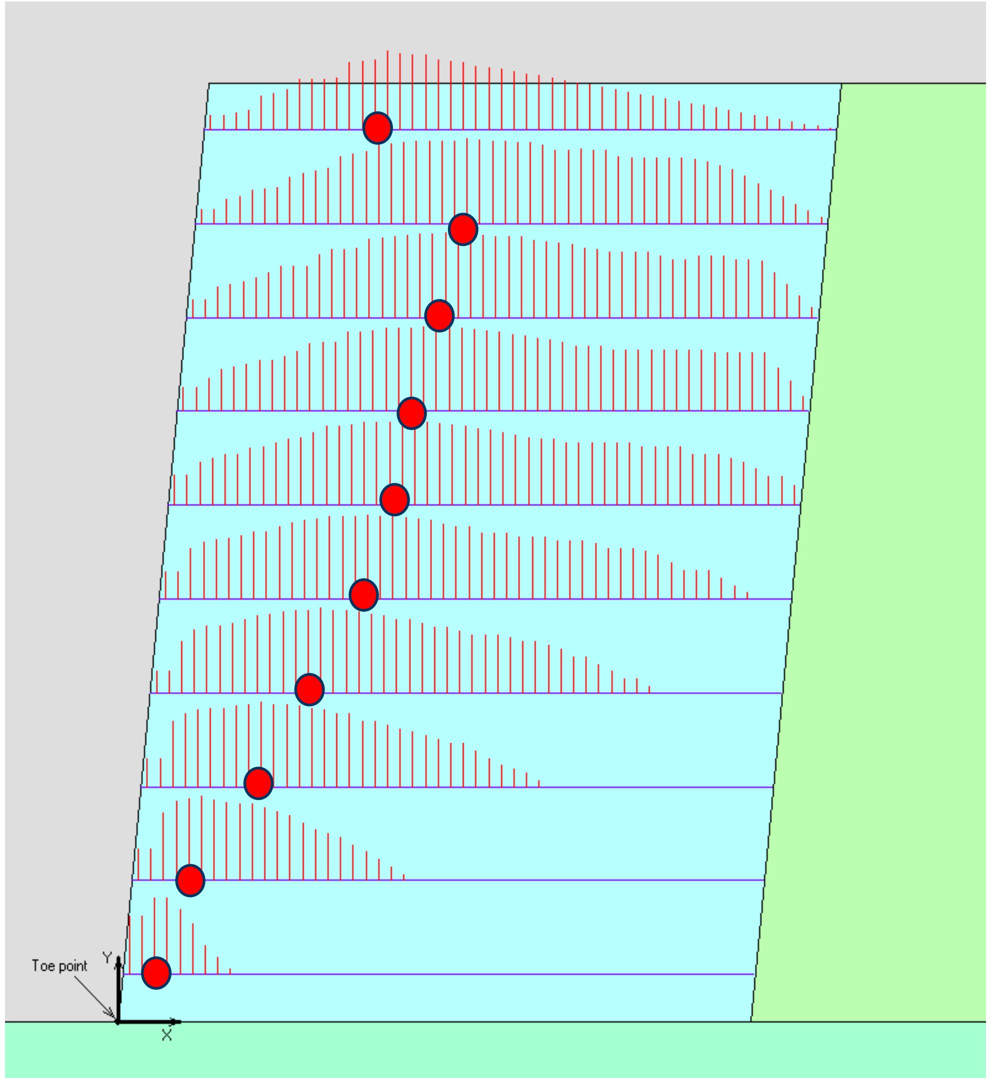


AASHTO assumes singular Rankine's wedge within reinforced mass ➔ Its inertia is *evenly* distributed amongst all layers to produce T_{md} ➔ AASHTO ignores compound-like wedges where fewer layers are carrying larger inertia loading

Effects of Facing: Small Blocks

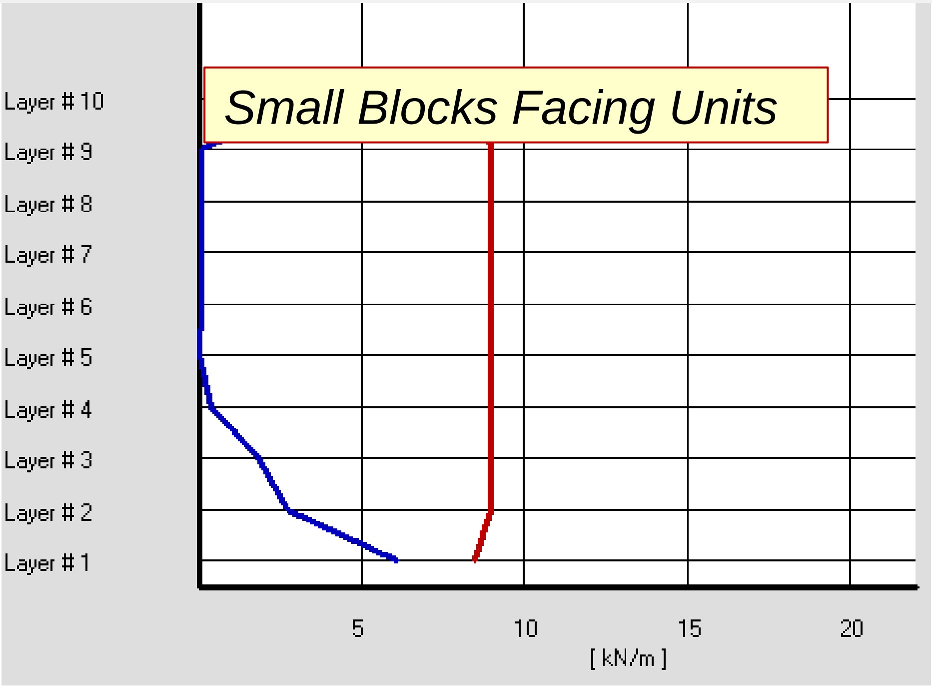


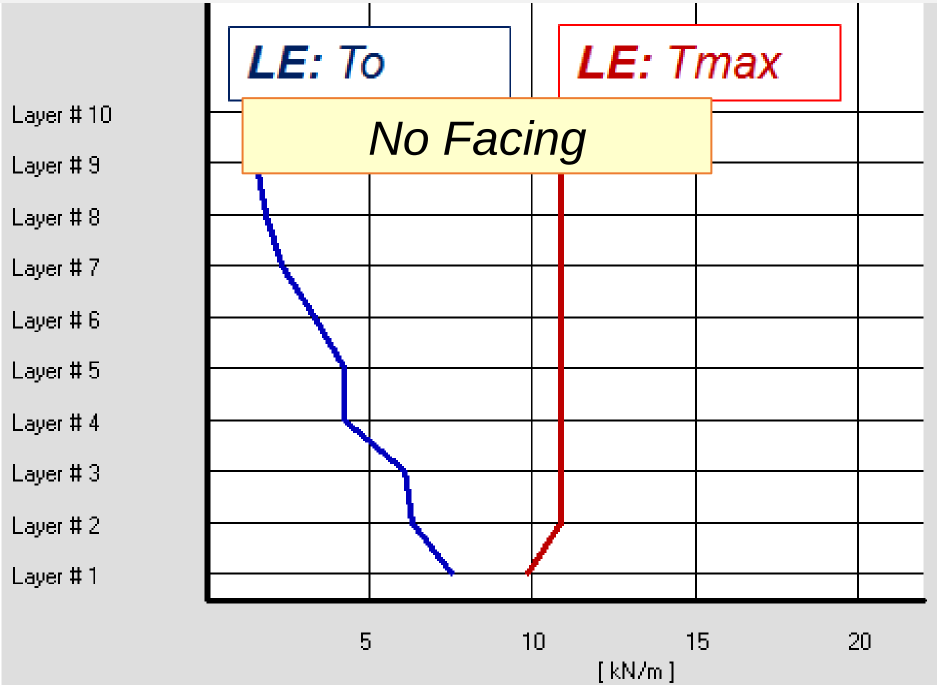
Blocks: γ_u=20 kN/m3; W_u=0.3 m;
H_u=0.20 m; 'c$_u$'=10 kPa & ϕ_u=30°



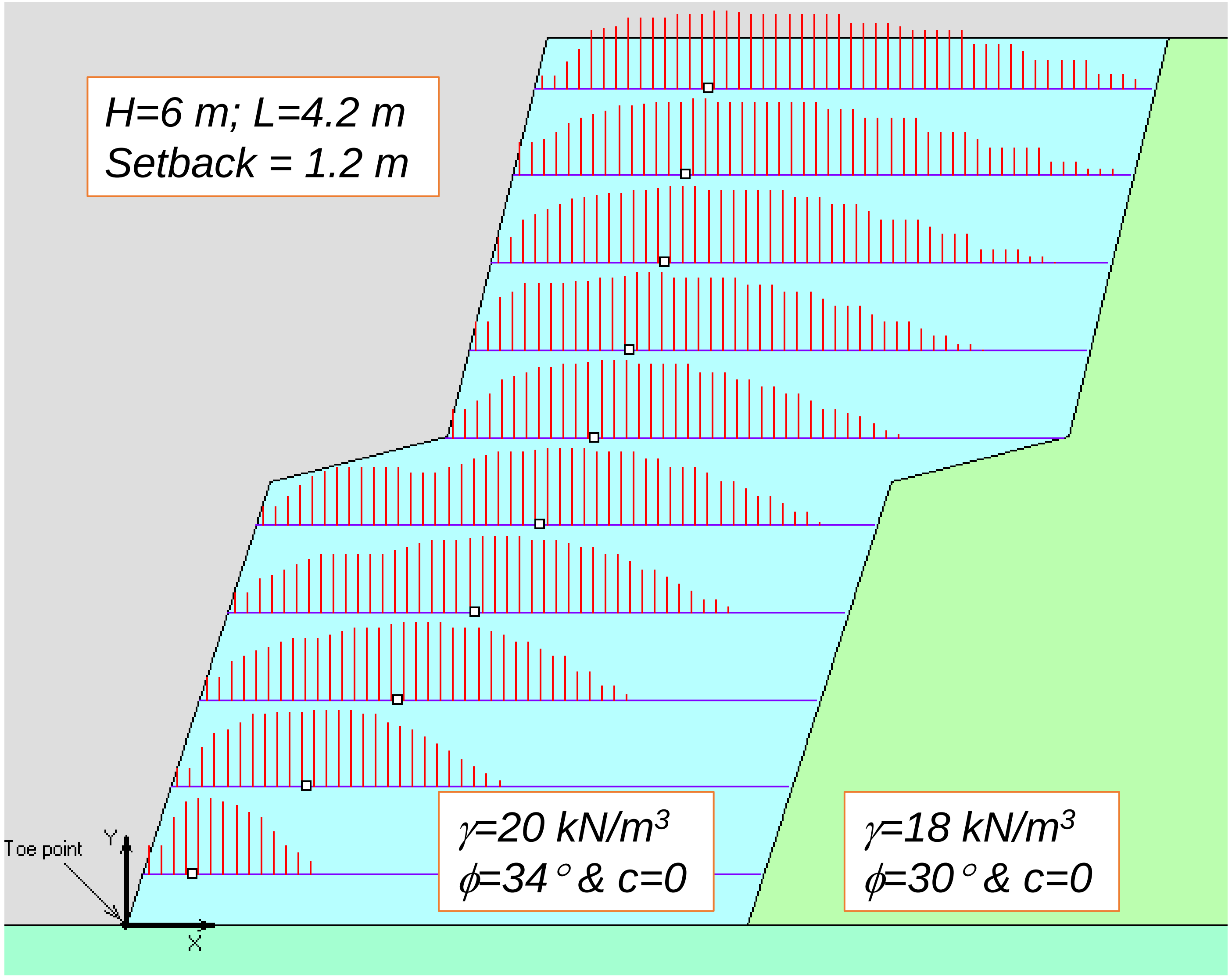
Effects of Small Blocks Facing: T_{max} and T_o

Large blocks or high interblock and toe resistance may reduce significantly the need for reinforcement (length and strength)

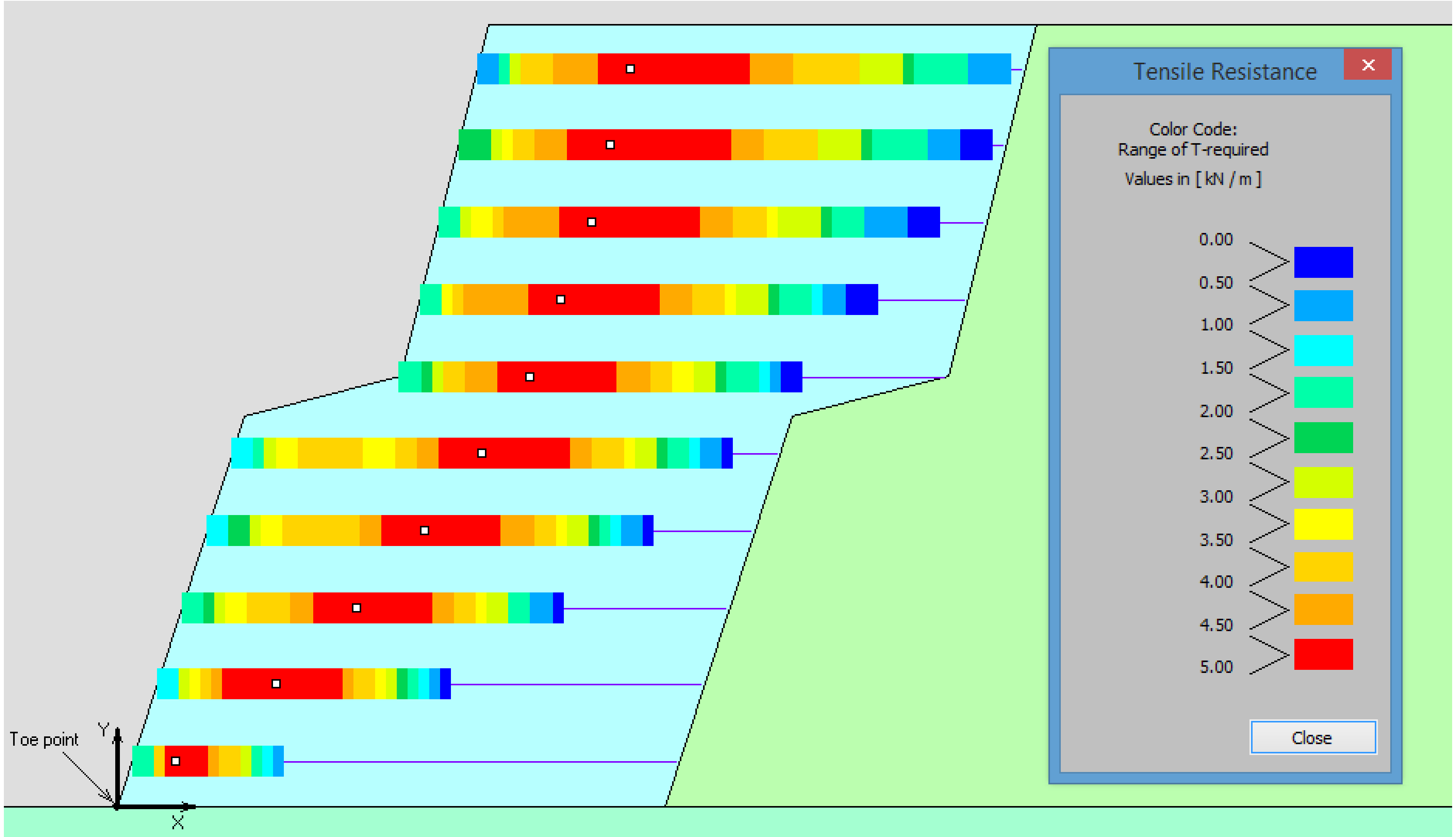




3(v):1(h) Two-Tier Wall



Tension Map: 2-Tier Wall



Roadmap of Presentation

- Why Limit State analysis is needed?
- Available Limit State Methods of Analysis
- Limit Equilibrium: Global Approach
- The Safety Map Tool
- Limit Equilibrium: Baseline Solution
- Limit Equilibrium: Design Approach
- Limit Equilibrium: Design Examples
- **Concluding Remarks**

Concluding Remarks

- Baseline Solution: Fs=1.0 on soil strength is used to determine LTDS, consistent with Internal Stability principles ➔ LRFD can be used, same as in AASHTO

- T_{max} and T_o: Global Stability ignores possible local overstressing while the Baseline Solution considers local demand rationally

- Global LE: Applicable to external stability -- sliding, eccentricity, and bearing load

Concluding Remarks - Reference

Explicit LE example, adjusted to LRFD framework per AASHTO 2020, is detailed in:

Geotechnical Fundamentals for Transportation Projects – Geotechnical Engineering Circular (GEC) No. 1

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